Number is Complex!

Perceptual Subitizing

You perceive the three or four dots intuitively & simultaneously.

You "just know."

Conceptual Subitizing

You perceive the parts & put together the whole

All of this happens quickly and often is not conscious - it is still subitizing

A *Big Idea:*

Quantity is an attribute of a set of objects

3 elephants ***might seem obviously bigger when compared to*** ***3 mice ...***

… if you used the attribute of size, but, for the attribute of number/ numerosity, they are identical.

A *Big Idea:*

Quantity is an attribute of a set of objects.

A Collection Can Have Many Attributes

Roses

- Red color is an attribute
- Round shape is an attribute
- Sweet smell is an attribute
- Quantity is another attribute: there are THREE roses in this collection.

A *Big Idea*

Quantity is an attribute of a set of objects.

We call this *numerosity* - the "threeness" of 3.

Numerosity exists apart from number words and written symbols. Words and symbols vary from language to language - numerosity does not.

Humans seem to be biologically programmed to automatically perceive the numerosity of small sets.

Counting,
determining & naming quantity,
telling "how many" …
all of these use number
as an adjective, not a noun.
Are there 3 dots? 3 fingers?
3 inches? 3 tenths?

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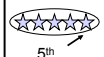
Big Ideas of Number Sense

Topic	Big Ideas	Examples
Uses of Number 	• Numbers are used many ways, some more mathematical than others.	• *Tommy has 5 books.* (cardinal) • *Ava is fifth in line today.* (ordinal) • Numbers on basketball jerseys, home addresses, telephone numbers (nominal) • *Let's meet at 5 pm on December 5.* (referential)
Numerosity  = 5	• Quantity is an attribute of a set of objects and we use numbers to name specific quantities. • The quantity of a small collection can be intuitively perceived without counting.	• 5 mice and 5 elephants are alike in quantity, though different in other ways. • Children just "see" three objects and know it's 3.

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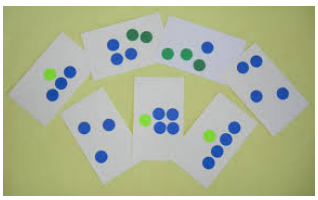
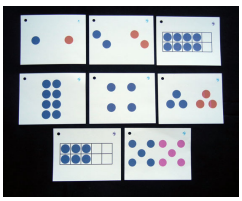
Developing Visual Number Sense Through Models





Dot Cards

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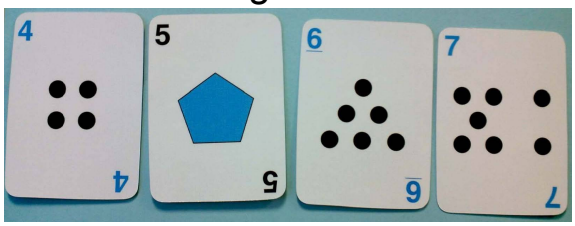
Developing Visual Number Sense Through Models

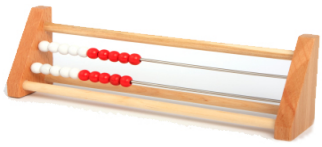


Number Cards

Developing Visual Number Sense Through Models

Dice

Developing Visual Number Sense Through Models

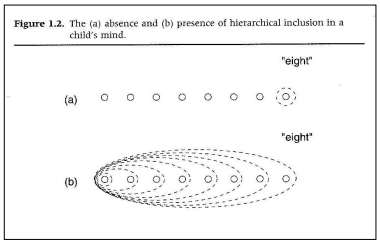


Counting Frames or Rekenreks

Why focus on developing number sense?

We want children to connect counting to cardinality and to use numbers meaningfully to solve problems

Hierarchical Inclusion
Because each number names a quantity one greater than the number before it, each number includes all the quantities named by those earlier numbers.



From Kamii,C. *Young Children continue to Reinvent Arithmetic, 2nd Grade.* Teachers College Press, 2004.

Solid number sense is key to **mathematically healthy** development & practice.

Number sense is not static. It continues to deepen & build as new relationships amongst more numbers are established.

How do children develop the idea of quantity as an attribute?

Older infants often learn signs/words for "more" and "all gone" before other ideas.

One-year-old can tell that a pile of 5 is more than a pile of 2, but does not know any number names.

In 2nd half of 2nd year (18-24 mos), toddlers can "take one" or "give two," but do not have words for 3 and bigger.

Preschoolers are building a firm sense of the numerosity of 3, 4 & 5.

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Implications for Teaching & Learning

Subitizing is foundational.

- Subitizing relies on visual patterns.
  - Not all arrangements of a number are equally easy to "see."
- How is subitizing different than counting?
  - Label small sets with number, without enumerating.
- How does subitizing support counting?
  - Authentic reasons to count small sets.
  - When counting, restate the last count word to emphasize cardinality: "1, 2, 3, … 3 cups."
- Subitizing & counting build understanding of cardinality.
- Expect children to subitize small sets; avoid "counting to be sure."

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More Implications for Teaching & Learning

Children learn about quantity even without exact numbers.

- "Which pile has more?"
- "Put one napkin on each plate."

Smaller numbers are easier than larger.

- With infants and toddlers, talk about "1" and "2" and "1 more" and "2 more"
- With preschoolers, spend a lot of time exploring "3" and "4" and "5"

Fingers are great tools for understanding small numbers, then building to 5 & 10.

Children need repeated exposure to amounts in order to associate number name and quantity.

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Focus on the Child: Number Sense

Questions to Consider While Viewing the Video Clips

- What Big Ideas about number do these children seem to understand?
- What Big Ideas about number do these children seem to be learning?
- What, specifically, do the children say or do that gives you evidence of their thinking?
- What opportunities for exploring number might a teacher provide for these children to encourage them to develop their understanding further?

Clip/Child		Notes about children's thinking
"Comparing Quantity"	*Child 4, preschool (1:52)*	
"Recognizing Quantity"	*Child 1, preschool (0:31)*	
	Child 2, preschool (0:27)	
"Matching Quantity"	*Child 3, preschool (1:29)*	

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Research Lesson: Number Arrangements

Some Questions to Consider While Viewing the Video Clip (6:02 total time)

Questions	Notes
<u>About the Children</u> ▪ What Big Ideas about number do these children seem to understand? ▪ What Big Ideas about number do these children seem to be learning? ▪ What, specifically, do the children say or do that gives you evidence of their thinking?	

<u>About the Teacher</u>
▪ What kind of instructional decisions has the teacher made in terms of the logistics of this activity?
  ○ About materials to use?
  ○ About questions to ask?
  ○ About space arrangement?
▪ How does the teacher scaffold the children's thinking and explaining?	

<u>About the Activity</u> • What does it do for the children's understanding to describe their arrangements using numbers? ▪ What modifications might you make if you were doing this activity in your classroom? ▪ What opportunities for exploring number might a teacher provide for these children to encourage them to develop their understanding further?	

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NUMBER ARRANGEMENTS

Big Idea Focus: Small collections can be intuitively perceived without counting (subitizing). Quantity is an attribute of a set of object.

In this lesson, children:

- Subitize small quantities of objects (1—5)
- Explore the possible arrangements for a given quantity of objects
- Describe a wide variety of number arrangements

Materials

- Manipulatives for arranging sets, such as craft sticks, cotton balls, connecting cubes, or inch tiles (for materials that come in multiple colors, use only one color at a time)
- Half-sheets of paper to display number arrangements
- Index card(s) labeled with the focus number and corresponding number of dots

1. Introduce Small Group Activity

Gather a small group of children around a table or in a circle on the rug. Arrange 3 craft sticks on a half-sheet of paper. Then ask children to arrange 3 craft sticks in a different design on half-sheets of paper. Point out all the different designs children find. Push the papers to the center of the small group to form a "Three Museum."

Invite children to continue exploring "three," making as many different arrangements as they can—one arrangement per half-sheet.

2. Discuss Number Arrangements

First, quickly check to be sure that all arrangements in the "Three Museum" contain the correct quantity. If some are not correct, ask children to count several arrangements, some with the correct quantity and some without, allowing children to discover the errors and correct them.

Then ask children to look at the various arrangements and describe what they see. At first, children will likely think of things the arrangements look like, such as:

- *It's a letter A.*
- *It's a triangle*
- *It's a box with no top* (|_|)

Planning Tips

This activity can last several weeks, with the focus on each number lasting several days. Once the activity is introduced to **small groups**, stations with different materials can be set up for center time.

All children, regardless of their apparent facility with abstract number, will benefit most from exploring number arrangements by beginning with 3, progressing to 4, and then to 5. Kindergarteners may be interested in extending this activity to 6 and 7.

Differentiation

For some children, you may want to hand them a set of 3 sticks, rather than focus on the counting out task.

Also, for the first experience, some children may benefit from copying one or more of your arrangements before trying to create their own designs.

Math Note

Do not require children to count every arrangement. The focus here is on discovering and gradually internalizing the unique patterns of each number. Children will come to ***subitize***—or intuit—the number they are exploring.

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Next, focus children on one arrangement.

- *If we wanted to describe this arrangement* (|_|) *with numbers, how might we do it?* (e.g., 2 and 1)
- *Are there other arrangements in the Three Museum that fit the same number pattern?*
- *Is there a different way we could describe this arrangement* (|_|)*?* (e.g., 1 and 1 and 1)

3. Continue and Extend the Explorations

Set up several stations with different manipulatives to explore the number three at center time. Each station should include only one type of manipulative. Label each station by writing the number 3 and three dots on an index card.

When children have explored three using several different materials and arrangements, they are ready to record their findings. If a station has a consumable material, such as craft sticks or cotton balls, children can glue down their arrangements on paper. For other manipulatives, children can record with drawings or you can take photos. These representations can be used for display or to make number books.

Continue this activity with four, and then five. Be sure your discussion of the number arrangements builds on previous explorations.

Book Connections

As your class begins exploring numbers, you may want to read aloud, and make available to children, the following books:

> *Anno's Counting Book* by Mitsumasa Anno

> *Ten Black Dots* by Donald Crews

> *1, 2, 3 to the Zoo* by Eric Carle

> Other counting books with good visual representations of numbers

Observation

*Are children able to **subitize** small quantities? That is, are they able to "see" small numbers instantly, without counting?*

Facilitation Tip

You might want to have children make a tally after their name on a duplicated class list to keep track of who has visited each station.

Differentiation

Avoid the temptation to "skip" children to bigger numbers. Different materials afford different discoveries about a number. And, the understandings children gain from exploring small numbers are the building blocks for later explorations.

If children need greater challenge, add a second color of manipulatives and ask them to find all the possible combinations of a number.

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