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INNOVATIONS

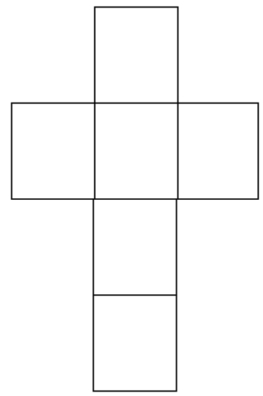
Learning Lab #10

Agenda

- Sorting Solids
- Investigation: Shape Nets
- Video Analysis
  - HIS-EM framework
  - CCSS for Mathematical Practice
- Geometric Thinking: van Hiele's levels
- Polygons: Triangle Talk
- Getting Good Math Out of Good Books

Creating a Net:
A Math Investigation for Adults

- We want to make boxes that are the same as these solids, but we do not have the designs we need. Please help!
- As a first step of our backwards design process, make a net of your solid shape.
- When you think you've done it, cut and fold to check.
- Here is a net for a cube:



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Describing 3-D Shapes:
*More Math Investigation **for Adults***

- On an index card, write a mathematical description of your 3-D shape (without using its name).
- After cards are shuffled, you'll get a new one. Try to find the shape that it describes.
- After you've found your match, gather with others who have the same type of shape to compare.

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Video Analysis: *"3-D Shapes in 2nd Grade"*

This is the activity table that the students in the video are using.

Edges, Vertices, and Faces

Look at each skeleton of a 3-D shape that you made. How many edges, vertices, and faces does each 3-D shape have? Record the number on the data table.

Skeletons of 3-D Shapes Data Table

Type of Shape	Number of Edges	Number of Vertices (Corners)	Number of Faces (Sides)
cube			
triangular prism			
rectangular pyramid			
triangular pyramid			
rectangular prism			

Video Analysis
"3-D Shapes in 2nd Grade"

Teacher Practice (HIS-EM): Concept Development

High-Impact Strategies

- Teachers ask questions that focus students on the structure of the problem.
- Teachers draw students' attention to the Big Ideas.

Student Practice (Common Core)

#7: Look for and make use of structure.

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What evidence of these TEACHER practices do you see in the video?
Are there missed opportunities for engaging in these practices?
What INSTRUCTIONAL DECISIONS has this teacher made?

HIS-EM dimension: Concept Development
High-Impact Strategies: **Teachers ask questions that focus students on structure of problem.** **Teachers draw students' attention to the Big Ideas.**
Does the lesson lead students to a deeper understanding of concepts? *Does the teacher help students generalize what they've learned?*

What evidence of these STUDENT practices do you see in the video?

Are there missed opportunities for engaging in these practices?

CCSS for Math Practice #7: **Look for & make use of structure.**
Does the lesson lead students to a deeper understanding of concepts? *Does the teacher help students generalize what they've learned?*

Development of Geometric Thought: The van Hiele Theory

	Levels of Thinking Processes	**How we think about shapes**	**Understanding of relationships between shapes**	**Implications for Instruction**
Elementary School ***PreK—3 Focus**	***Level 0**: Visualization	Think about shapes holistically A triangle is a triangle because “it looks like a triangle” Properties are not abstracted from the shape	Can create groupings of shapes that “look alike” Appearances can be distracting, so may not recognize all triangles (right, obtuse, etc.) as triangles	Provide many opportunities for sorting and classifying shapes Introduce language for the geometric properties of shapes (# of sides and corners, symmetry, etc.) Provide a wide variety of shape examples (and non-examples)
	***Level 1**: Analysis	Notice the properties that make up geometric shapes, and lists as many as they know Can compare, measure, sort, and describe shapes by properties Think about classes of shapes (*all* rectangles, not just *this* rectangle)	Generalize ideas about an individual shape to all shapes that fit that class Does not see the relationships between classes of shapes (e.g. all squares are rectangles, and all rectangles are parallelograms)	Focus more on properties of shapes that on simple identification Ask children to apply ideas to entire classes of shapes (*all* rectangles, *all* prisms, etc.)
	Level 2: Informal Deduction	Use “if-then” reasoning to classify shapes using a minimum set of properties (e.g. rectangles are parallelograms with a right angle)	Appreciate the logic of hierarchical classification schemes	Ask children to define shape classifications by simple chains of reasoning (informal proofs) Focus on the properties that guarantee a shape, rather than a complete list of properties
High School	**Level 3**: Deduction	Define geometric objects and properties (formal proofs)	Use a system of logic to define relationships among shapes from which other truths can be derived	N/A
College & beyond	**Level 4**: Rigor	Formal axiomatic geometry	Study relationships among different axiomatic systems of geometry	N/A

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van Hiele's Levels of Geometric Thinking: Implications for Teaching

- The levels are sequential.
  - The thinking and understanding of prior levels are required for further development.
- The levels are not age dependent.
  - Geometric experience is the greatest single factor influencing advancement.
- The use of physical materials, drawings, and computer models are important at every level.
- When instruction is at a level higher than that of the students, learning will likely be temporary and superficial.

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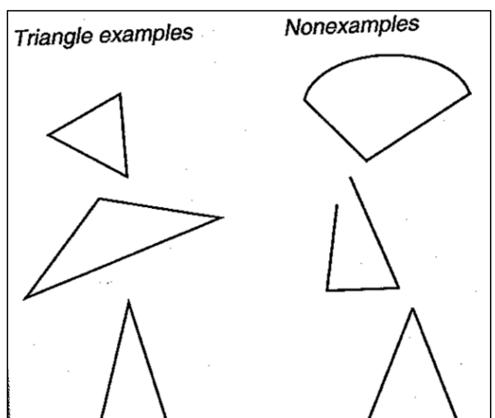
To give students a chance to think precisely about shapes, it is important to show a wide variety of examples & non-examples.



Triangle examples & non-examples reprinted from Copley, J. *The Young Child and Mathematics, 2nd ed.*, p.107 (NAEYC & NCTM, 2010).

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"December - When is a triangle a triangle?' Reprinted from *Little Kids - Powerful Problem Solvers: Math Stories from a Kindergarten Classroom* by A.G. Andrews & P. R. Trafton (Heinemann, 2002).

Readers Theater Roles
Narrator/Teacher
Cameron
Elana
Rachel
Kevin
Starla
Elizabeth

The Lesson

I stood next to my overhead projector and called the children to come and sit around it. I showed the scene that was depicted and discussed what was in it. Then I said, "Our job today is to find all the triangles in this picture. Who can find one?"

Many hands went up. A child came and outlined the pennant on the boy's bike (an isosceles triangle). Clearly this child didn't view a triangle as having three equal sides with one of them horizontal. Assuming the children understood the task, I was ready to move on to the next shape. However, the children stopped me. Their waving hands indicated they were enjoying doing the activity together and wanted to keep on going. So I called on a few more children. The next child outlined the top of the traffic light (an equilateral triangle). The triangle in the *A* on the pizza sign was outlined by another child. Then Cameron outlined a yield sign and called it an upside-down triangle.

What Is a Triangle?

"Does everybody agree that this is an upside-down triangle?" I asked.

"Oops!" Cameron said. "Triangles have three sides—they can go any direction. I just forgot." I was glad he remembered our emphasis on this earlier in the year when I had shown a triangle shape in many positions, each time asking, "Is this a triangle now?" I had wanted the children to know that position did not influence whether something was a triangle.

Another child came to outline a slice of pizza on a sign. I was ready to move on, so I asked, "Now, who can find a circle?"

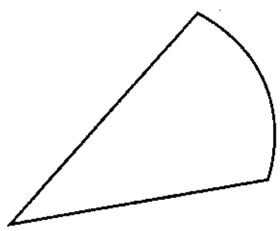


FIGURE 5–1 Pizza Slice

"Wait!" Elana's hand was up and waving. "There is another triangle!" She came up and outlined a shape that was the side of a traffic light. (See Figure 5–2.)

"Why do you say this is a triangle?" I asked.

"Because it has three corners," Elana replied.

I then realized that the worksheet had mixed drawings of triangles with real-world objects that resembled triangles, but were not triangles. I asked, "Does everyone agree that this is a triangle?"

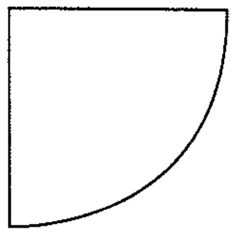


FIGURE 5–2 Traffic Light—Side View

Many children agreed, but some were uncertain.

"It has three corners but it doesn't really look like a triangle," Rachel said.

"Does anyone else think this isn't a triangle?" I asked.

"It really might not be a triangle," Kevin said. He came to the overhead and pointed to the shape. "This side [pointing to the curved side] looks like a half-moon shape. I don't think triangles look like this."

"Well," Starla said, "it has three sides. The pizza slice has that kind of side, too, and it's a triangle."

"Then I don't think it is a triangle, either," Kevin argued.

Oops—in my hurry to move along, I hadn't noticed that the pizza slice was not a triangle.

The children seem confused. Many thought Elana's shape was a triangle because it had three corners and three sides. Others were not sure.

"Well, what about that sign," said Cameron, pointing to the yield sign he had circled earlier. "It has curvy parts on the corners, too." (See Figure 5–3.)

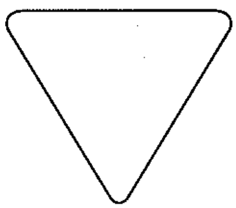


FIGURE 5–3 Yield Sign

"Can triangles have curvy parts?" Elizabeth asked.

A discussion broke out among the children. It centered on whether triangles could have curvy parts and whether their corners had to be "pointy." I let them talk a bit while I gathered my thoughts and listened to theirs.

"Let me mention something else about triangles," I told them. "You were right that triangles must always have three corners and three sides, but the sides also have to be straight. Does this help you with this problem?"

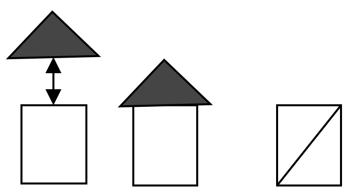
"Oh, then I think the pizza slice is not a triangle and this one isn't, either," said Kevin, pointing to the quarter circle shape.

"Neither is the yield sign!" Cameron said.

"Three straight sides," Elizabeth said, to nobody in particular.

> Students need to see many examples of shapes that correspond to the same geometrical concept as well as a variety of shapes that are non-examples of the concept. For example, shapes that have a resemblance to triangles, but are not triangles.
>
> —*Principles and Standards for School Mathematics* (98)

Big Ideas of Shape

Topic	Big Ideas	Examples
Defining & Analyzing Shapes 	●Shapes can be defined and classified by their attributes. ●The flat faces of solid (three-dimensional) shapes are two-dimensional shapes. ●Shapes can be combined and separated (composed and decomposed) to make new shapes.	•A rectangle must have two sets of parallel sides of equal length and four 90º angles; thus, a square is a special type of rectangle. •A baseball is a sphere and can be represented in a drawing as a circle. •Any rectangle can be divided into 2 triangles.

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Books to Inspire Children's Mathematical Thinking About Shapes & Space

Burns, M. *The Greedy Triangle.* (Scholastic, 1994)

This silly story introduces polygons in real-world contexts; math notes for adults at the back.

Carle, E. *The Secret Birthday Message.* (HarperTrophy, 1971)

Relative position words, shape symbols, and a map in a beautifully illustrated story about finding a treasure.

Ehlert, L. *Color Farm.* (HarperCollins, 1990)

Ehlert, L. *Color Zoo.* (HarperCollins, 1989)

Bright, bold, cut-out pictures illustrate shape composition in the context of animal faces.

Greene, R.G. *When a Line Bends … A Shape Begins.* (Houghton Mifflin, 1997)

Rhyming text describes shapes in the world (though not with mathematical precision).

Hall, M. *perfect square.* (Greenwillow, 2011)

A story of a square that is used to make various pictures, before becoming a square window – vivid pictures!

Hoban, T. *Cubes, Cones, Cylinders & Spheres.* (Greenwillow, 2000)

Hoban, T. *over, under & through.* (Simon & Schuster, 1973)

Hoban, T. *Shapes, Shapes, Shapes.* (Mulberry, 1986)

All of Hoban's books have no (or few) words, but stunning photos (many of urban life) that can spawn discussion.

Hutchins, P. *Rosie's Walk.* (Aladdin, 1968)

This simple, fun story illustrates a variety of relative position words.

Jenkins, S. *Looking Down.* (Houghton Mifflin, 1995)

The bright collage pictures in this wordless book introduce the idea of scale and can lead to rich math talk.

Maccarone, G. *Three Pigs, One Wolf, and Seven Magic Shapes.* (Scholastic, 1997)

Tangram puzzles are the key to the little pig put-witting the wolf.

Micklethwait, L. *I Spy Shapes in Art*. (Greenwillow, 2004)

Find shapes in famous works of art.

Neuschwander, C. *Sir Cumference and All the King's Tens.* (Charlesbridge, 2009)

Neuschwander, C. *Sir Cumference and the Dragon of Pi.* (Charlesbridge, 1999)

Neuschwander, C. *Sir Cumference and the First Round Table.* (Charlesbridge, 1997)

Neuschwander, C. *Sir Cumference and the Great Knight of Angleland.* (Charlesbridge, 2001)

Neuschwander, C. *Sir Cumference and Isle of Immeter.* (Charlesbridge, 2006)

Neuschwander, C. *Sir Cumference and the Sowrd in the Cone.* (Charlesbridge, 2003)

Neuschwander, C. *Sir Cumference and the Viking's Map.* (Charlesbridge, 2012)

All of the *Sir Cumference* books are complex medieval adventures where mathematical thinking saves the day!

Onyefulu, I. *A Triangle for Adaora.* (Frances Lincoln, 2000)

Bright photos illustrate this text-full story of a child who helps a younger sibling find shapes.

Pilegard, V.W. *The Warlord's Puzzle.* (Pelican, 2000)

A richly illustrated story of one possible origin of tangram puzzles.

Pluckrose, H. *Shape: Math Counts*. (children's Press, 1994)

Photographs show shapes in real-world contexts, including when the same object appears to have different shape.

Tompert, A. *Grandfather Tang's Story.* (Crown, 1990)

This "tale told with tangrams" involves many puzzles and can inspire tangram-tale-telling in primary students.

Rosen, M. & Oxenbury, H. *We're Going on a Bear Hunt.* (Aladdin, 1989)

This classic rhyme provides a fun way to act out relative positional words.

Sweeney, J. *Me on the Map.* Dragonfly Books, 1996)

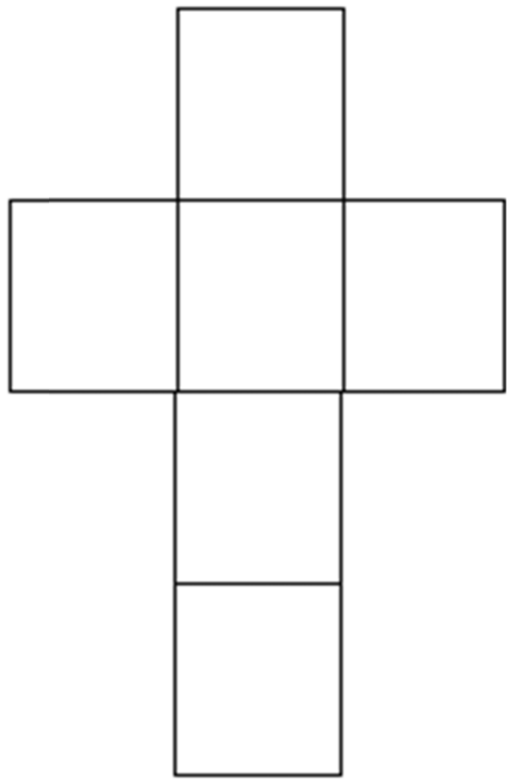
Shows how maps are special kinds of pictures; simple language, but complex ideas about perspective and scale.

Walsh, E. S. *Mouse Shapes.* (Harcourt, 2007)

The mice combine shapes to trick the cat.

A Geometric Puzzle to Ponder

This is a net for a cube:



How many other, different, nets for a cube are there?