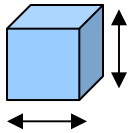
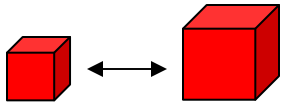
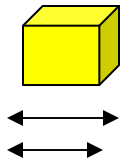
erikson
early mathematics education
INNOVATIONS

Learning Lab #7

Agenda

- *Begin with Breakfast*
- Investigation: Ordering Objects
- Exploring Comparison: *The Long and Short of It*
    - Video Analysis
- Considering Precision
- Big Ideas of Measurement
- Rulers
- CCSS for Mathematical Practice
- *Break for Lunch*
- Number Line as Measurement Model
    - Video Analysis
- Linear Measurement Learning Trajectory
- Constructing a Conceptual Understanding of Measurement
- High-Quality Books to Spark Mathematical Thinking & Action

Big Ideas about Measurement

Topic	Big Ideas	Examples
Attributes 	• Many different attributes can be measured, even when measuring a single object.	• A bucket has many measurable attributes, including height, weight, capacity, or circumference: *What kind of "big" is it?*
Comparison 	• All measurement involves comparison. • To be accurate, measurement must be "fair."	• Weighing rocks on a pan balance (direct); using a length of string to measure a table in one room and chairs in another (indirect). • A fair comparison measures the same attribute. **Units** must be of equal size, with no gaps or overlaps.
Precision 	• Quantifying a measurement helps us describe and compare more precisely.	• Nonstandard **units** (such as blocks) and standard **units** (such as inches) allow for more precision than direct comparison. • There is always a more precise measurement possible – we never get it exactly "right," but it must be "good enough."

erikson
early mathematics education

Major support for the Early Mathematics Education Project is provided by tMcCormick Foundation, CME Group Foundation , U.S. Dept. of Ed. Investing in Innovation Fund (i3), Motorola Foundation, CPS Office of Early Childhood Education, Exelon Corporation & Robert & Isabelle Bass Foundation

Wally's Stories

Conversations in the Kindergarten

VIVIAN GUSSIN PALEY

"A vivid and credible picture
of how five-year-olds think."
—D. J. Enright, *Times Literary Supplement*

Harvard University Press © 1981

Rulers

Rulers were another example of the wide gulf separating my beliefs from those the children demonstrated whenever they were allowed to follow their ideas to logical conclusions. I had not realized that "rulers are not really real." We were about to act out "Jack and the Beanstalk" when Wally and Eddie disagreed about the relative size of our two rugs.

Wally: The big rug is the giant's castle. The small one is Jack's house.

Eddie: Both rugs are the same.

Wally: They can't be the same. Watch me. I'll walk around the rug. Now watch—walk, walk, walk, walk, walk, walk, walk, walk, walk—count all these walks. Okay. Now count the other rug. Walk, walk, walk, walk, walk. See? That one has more walks.

Eddie: No fair. You cheated. You walked faster.

Wally: I don't have to walk. I can just look.

Eddie: I can look too. But you have to measure it. You need a ruler. About six hundred inches or feet.
Wally: We have a ruler.
Eddie: Not that one. Not the short kind. You have to use the long kind that gets curled up in a box.
Wally: Use people. People's bodies. Lying down in a row.
Eddie: That's a great idea. I never even thought of that.

Wally announces a try-out for "rug measurers." He adds one child at a time until both rugs are covered—four children end to end on one rug and three on the other. Everyone is satisfied, and the play continues with Wally as the giant on the rug henceforth known as the four-person rug. The next day Eddie measures the rugs again. He uses himself,Wally, and two other childen. But this time they do not cover the rug.

Wally: You're too short. I mean someone is too short. We need Warren. Where's Warren?
Teacher: He's not here today.
Eddie: Then we can't measure the rug.
Teacher: You can only measure the rug when Warren is here?
Jill: Because he's longer.
Deana: Turn everyone around. Then it will fit.
(*Eddie rearranges the measurers so that each is now in a different position. Their total length is the same.*)
Eddie: No, it won't work. We have to wait for Warren.

Deana: Let me have a turn. I can do it.
Jill: You're too big, Deana. Look at your feet sticking out. Here's a rule. Nobody bigger than Warren can measure the rug.
Fred: Wait. Just change Ellen and Deana because Ellen is shorter.
Jill: She sticks out just the same. Wait for Warren.
Fred: Now she's longer than before, that's why.
Teacher: Is there a way to measure the rug so we don't have to worry about people's sizes?
Kenny: Use short people.
Teacher: And if the short people aren't in school?
Rose: Use big people.
Eddie: Some people are too big.
Teacher: Maybe using people is a problem.
Fred: Use three-year-olds.
Teacher: There aren't any three-year-olds in our class.
Deana: Use rulers. Get all the rulers in the room. I'll get the box of rulers.
Eddie: That was *my* idea, you know.
Deana: This isn't enough rulers.
Eddie: Put a short, short person after the rulers—Andy.
Andy: I'm not short, short. And I'm not playing this game.
Wally: Use the dolls.
Teacher: So this rug is ten rulers and two dolls long? (*Silence.*) Here's something we can do. We can use one of the rulers over again, this way.

Eddie: Now you made *another* empty space.
Teacher: Eddie, you mentioned a tape measure before. I have one here.
(*We stretch the tape along the edge of the rug, and I show the children that the rug is 156 inches long. The lesson is done. The next day Warren is back in school.*)
Wally: Here's Warren. Now we can really measure the rug.
Teacher: Didn't we really measure the rug with the ruler?
Wally: Well, rulers aren't really real, are they?

Rulers are not real, but rug measurers are. Dressing up to look like a mother and using magic to become a lion is real, but having parents die is not real. Does "real" mean that which can be imagined and acted out? Does Wally *see* himself as a mother lion rather than expect to *be* one? Wally once told Eddie he was going to grow up and become Superman. "You can't do that, Wally," Eddie said. Whereupon Wally altered his statement to "I mean *look* like Superman," and Eddie approved.

I discovered that the scale I had just paid twenty dollars for was no more real than the tape measure. We were about to act out *Stone Soup*, by Marcia Brown, a story about three hungry soldiers who trick some selfish peasants into giving them food by pretending to make soup out of three stones. As part of the play, the children brought vegetables to cook.

"Do stones melt?" Rose suddenly asked. "Do we eat the stones?"

"Do you think they melt, Rose?"

"Yes."

I looked around at serious faces. "Does anyone agree with Rose?"

"They *will* melt if you cook them," said Lisa.

"If you *boil* them," Eddie added.

No one doubted that the stones in the story had melted and that ours too would melt.

"We can cook them and find out," I said. "How will we be able to tell if they've melted?"

"They'll be smaller," said Deana.

I lower three stones into boiling water. "How long shall they boil?" I ask. The suggestions range from a few minutes to ten hours. We decide on one hour and finish the story while the stones cook. Just before lunch we remove the stones and place them on a table.

Ellen: They're much smaller.
Fred: Much, much. Almost melted.
Rose: I can't eat melted stones.
Teacher: Don't worry, Rose. You won't. But I'm not convinced they've melted. Can we prove it?
Mickey: Draw a picture of them.
Teacher: And cook them again? All right.
(*Mickey and Earl trace the stones on a piece of paper, and I put them back in the water to cook some more. Thirty minutes later the stones* do *look smaller.*)
Teacher: I know they seem smaller, but it's very hard to match stones and patterns. Is there another way to prove whether the stones have melted?
(*There is no response. Clearly I am after the "right" answer, but the children have enough proof that the stones have melted.*)
Teacher: Let's weigh them on this scale. How much do they weigh?

Everyone: Two.
Teacher: Two pounds.
Lisa: Do we have to cook them again? They'll just keep melting.
Teacher: Maybe not.
(*After a short period we weigh the stones again.*)
Eddie: Still two. But they *are* smaller.
Wally: Much smaller.
Teacher: They weigh the same. Two pounds before and two pounds now. That means they didn't lose weight.
Eddie: They only got a *little* bit smaller.
Wally: The scale can't *see* the stones. Hey, once in Michigan there were three stones in a fire and they melted away. They were gone. We saw it.
Deana: Maybe the stones in the story are magic.
Wally: But not these.

The endless contradictions did not offend them; the children did not demand consistency. Once Lisa told us that she and her family did not believe in the tooth fairy. Her mother gave her a quarter for her tooth. I asked what her mother would do with the tooth and why it was worth a quarter to her. "She can sell it to the tooth fairy and get real gold for it." Lisa saw nothing inconsistent about combining both ideas.

Nor did anyone at Lisa's table think it strange when she asked me if I were really Mrs. Paley. I had spoken of Mr. Paley during lunch and Lisa was surprised. "Then are you really Mrs. Paley?"

"Lisa, you know that's my name," I said.

"Yes," she replied, "but I thought you just called yourself that."

From Barlow & McCrory (2011), "3 strategies for promoting math disagreements," *Teaching Children Mathematics*,17(9): 530-539.

FIGURE 4

Such tasks as using a broken ruler for measuring are designed to uncover misconceptions and elicit disagreements.

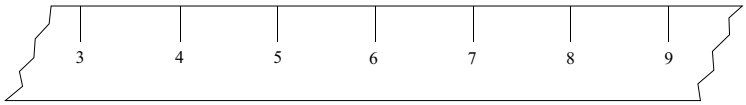


FIGURE 5

Had disagreements between students not occurred, gaps within their understanding most likely would not have been discovered. As students demonstrate their viewpoints to their peers, they make connections between their ideas, and they resolve other false ideas. (Teacher prompts are in bold text.)

[*Frank*] There's two answers.

Is it possible for something to have two different lengths? Outside of a piece of gum that can be stretched . . . If you can't stretch it, is it possible for something to have two different lengths?

[*Jenaria*] Like what Brianna said; like you can either round, and it can go up to six inches, or you don't have to round. You can just leave it like it is at five-and-a-half inches, or you can round it and have it be six inches.

[*Karl*] Yes, if you do it like that. [*He motions to two sides of the index card, pauses, then clarifies that the students were just measuring the long side of the card.*] Oh, no. Because the ruler only can be like [*pauses*] if you do it like that, you've still got the same thing [*pointing toward the numbers on the ruler*]. Only if you turn it around.

So that changes the length of the card when you move the ruler.

[*Karl*] Not exactly.

[*Using the broken ruler to demonstrate at the front of the class, Frank aligns the end of the index card with the end of the ruler.*] Now, there's 1, 2, 3, 4, 5, and a half. [*Afterward, he slides the index card so that it aligns with the three-inch mark.*] But now, um, there is 1, 2, 3, 4, 5, 6. [*Note that Frank is counting the inch marks on the ruler and not the spaces between them.*] You didn't need to move the ruler, because this is in centimeters.

[*Karl*] [I disagree with] him doing the centimeters. We are not learning about centimeters. That's mostly the only way you can do it two times. That's the only way you can get two different measurements—using centimeters and inches.

[*Ayona*] If he changes the ruler to here [*aligning the edge of the index card with the edge of the broken ruler*], then you have to count this part too [*motioning to the initial part of the index card before the three-inch mark*].

[*Frank*] Yes, I was counting this [*pointing toward the part of the card after the final inch mark*]. It's a half.

[*Ayona*] But you weren't counting this [*pointing toward the initial part of the index card before the three-inch mark*].

[*Frank*] Oh, yeah.

2. Reveal students' misconceptions

Tasks that are designed to address students' misconceptions provide the opportunity for disagreements to arise. As an example, consider the broken-ruler task, which is intended to reveal students' misconceptions regarding measurement of length. In this task, students were given a paper ruler (see **fig. 4**) and asked to use it to measure the long side of an index card. Having read about using a broken ruler (Barrett et al. 2003), we anticipated that the task would lead to a disagreement surrounding the index card's length, as some students would report its actual length (five inches), but others would misread the ruler. As the lesson unfolded, three separate disagreements arose. First, some students reported that the index card measured five-and-a-half inches *and* six inches, leading to a disagreement over whether the long side of the index card could have two different lengths (see **fig. 5**). Second, as students provided their arguments regarding the possibility of two different lengths, a disagreement arose over whether to count the lines on the ruler or the spaces between the lines. Finally, a disagreement surfaced over where to line up the index card, either at the initial line or at the end of the ruler.

Examining students' responses to this example reveals that using a task designed to uncover their misconceptions prompts disagreements that give students the chance to share their ideas and defend their reasoning. Student discussions generated by the disagreement reveal an interesting possibility: When measuring lengths, students can actually have different results if dissimilar units are used in the measuring process. According to Karl, "That's the only way you can get two different measurements—using centimeters and inches." Such a revelation facilitated thinking about the underlying assumption of the disagreement, that we were all using the same units.

Name_____

Length Knowledge Learning Trajectory

TABLE 1
This hypothetical learning trajectory for length knowledge is adapted from Clements and Sarama (2009).

Level	Observable operations	Mental actions on objects
End-to-end measurer (E to E)	Compares a train of short objects to an object. May use incongruent objects.	Expects that length is a composition of shorter lengths.
Unit relater and repeater (URR)	Composes length by combining parts. Attends to unit size. Composing, often not reversibly.	Iterates a mental unit along a perceptually available object. Allows for *counting-all* addition schemes.
Length measurer (LM)	Keeps identical units. Attends to the zero position. May partition units. Can integrate partitioning actions and grouping actions. Employs the broken ruler.	Well-developed scheme for linear composition and partitioning from units to *composite units* and also *units of units* of length. Understands that units are subdivisible.
Conceptual measurer	Can accurately estimate length, including a system of subunits. Operates with flexible arithmetic on collections of objects. Can coordinate operations among figures.	Has an interiorized length scheme that enables the child to mentally partition into a given number of parts, or project iterations onto objects. Enacts a multiplicative scheme to operate on units of units of units.

Getting Good Math out of a Good Book

erikson
early mathematics education

Title & Author: _____

Where's the Math? What's the Math that's There?

Analyze and discuss the book to identify specific BIG IDEAS of math that this book might be well suited to use to introduce or develop understanding for the children in your classroom. Please be as specific as possible. (For example, instead of stating "number sense" identify "composing and decomposing numbers up to ten," "reinforcing one to one correspondence," "understanding 2-digit place value," or …)

Big Ideas in this Book

What are some good open-ended discussion questions or problem situations I can ask that will trigger mathematical thinking or understanding?

(For example, every monkey in Caps for Sale has one hat. Which picture makes it easier to count how many - the one of the monkeys in the tree or the one of the man sleeping with his hats piled on his head? Why?]

What are some extending activities that will allow children to develop and construct mathematical understanding?

(For example, after reading Five Creatures, have children decide how many creatures live in their house and then draw and label a picture modeled after one in the book - collect into a classroom book.)

Key Features of the Book

(For example: Is the book a narrative? Is the focus informational? Is the art realistic, photo, or fanciful? Does the story feature human or animal characters? How do the illustrations and the text interact?)

Which statement best describes this book?

☐ Math concepts are the focus of the book rather than a story as such—counting books, books about shapes or fractions are common math concept books. (ABCs, colors & other "concepts" are often made into books as well.)

☐ Math situation/problem is embedded in the story (ex: *Roll Over, Grandfather Tang*)

☐ Math understandings/problems can be drawn out of the book (ex. *Caps for Sale*)

Mathematizing the Book *(with your class in mind):* What connections can you make?

Math to self (Is there a possible relationship between the math in book/story and kids' experience)

Math to math (Is the math situation related to a type of math problem situation in another book/story or to the math that you have been working on in math class?)

Math to world (Does the math in book/story relate to a real world issue or situation?)

To help you get going, here are some questions you might ask as you evaluate a counting book:

- *How high does the book count to? (1-10, higher)*
- *As the numbers change, are the illustrations additive (more join or leave the same group) or is each number a new set?*
- *If involving higher numbers, does the book introduce patterns or arrays or somehow reinforce the idea of place value?*
- *Does the book introduce separating and joining concepts (addition and subtraction)? Does it do so at fairly simple level of counting up (1-10, counting back (10 to 1) or both (10 Apples On Top does both). Is the addition and subtraction more complex? (Ann Jonas' Splash) Does the book use counting on?*
- *Is the counting embedded in a story that helps make a math-all-around-us connection?*
- *Is the counting tied to another informational concept, such as animal study?*

erikson
early mathematics education

Books to Inspire Children's Mathematical Thinking About Measurement

Aber, L. *Carrie Measures Up*. Kane Press, 2001.

Alborough, J. *Tall.* Candlewick Press, 2005.

Alborough, J. *Where's My Teddy?* Candlewick Press, 1992.

Aylesworth, J. *The Mitten.* Scholastic Press, 2009. [Alternate versions of story by Brett & Tresselt.]

Baker, K. *Just How Long Can a Long String Be?* Arthur A. Levine Books, 2004.

Brett, J. *The Mitten.* G. P. Putnam's Sons, 1989. [Alternate versions of story by Aylesworth & Tresselt.]

Briggs, R. *Jim and the Beanstalk.* Scholastic, 1970.

Burns, M. *Spaghetti and Meatballs for All!* Scholastic, 1997.

Carle, E. *The Grouchy Ladybug.* HarperCollins, 1977.

Carle, E. *Papa, Please Get the Moon for Me.* HarperCollins, 1986.

dePaola, T. *Strega Nona.* Aladdin, 1975. [Alternate versions of story by DeSpain & Ziefert.]

DeSpain, P. *The Magic Pot.* August House Story Cove, 2007. [Alternate versions of story by dePaola & Ziefert.]

Ehlert, L. *Un Lazo a la Luna / Moon Rope*. Harcourt Brace Jovanovich, 1992.

Florian, D. *A Pig is Big.* HarperCollins, 2000.

Hoban, T. *is it larger? is it smaller?* Greenwillow, 1985.

Jenkins, S. *Actual Size.* Houghton Mifflin, 2004.

Jenkins, S. *Big & Litlle.* Houghton Mifflin, 1996.

Kalan, R. *Blue Sea*. Mulberry Books, 1979.

Krauss, R. *The Growing Story*. HarperCollins, 2007/1947.

Leedy, L. *Measuring Penny*. Square Fish, 1997.

Lionni, L. *Inch by Inch.* HarperTrophy, 1960.

Mosel, A. *Tikki Tikki Tembo*. Henry Holt, 1968.

Myller, R. *How big is a Foot?* Yearling, 1962.

Pinczes, E.J. *Inchworm and a Half.* Scholastic, 2001.

Rathmann, P. *10 Minutes till Bedtime.* Puffin, 1998.

Rockliff, M. *Next to an Ant.* Children's Press, 2004. [También en español: *Al lado de una hormiga.*]

Rockwell, A. *One Bean.* Walker and Company, 1998.

Russo, M. *The Line Up Book*. Greenwillow Books, 1986.

Sweeney, J. *Me and the Measure of Things.* Dell Dragonfly Books, 2001.

Tompert, A. *Just a Little Bit.* Houghton Mifflin, 1993.

Tresselt, Al.R. *The Mitten*. Mulberry Books, 1964. [Alternate versions of story by Aylesworth & Brett.]

Walsh, E.S. *Balancing Act.* Beach Lane, 2010.

Wells, R.E. *Is a Blue Whale the Biggest Thing There Is?* Albert Whitman, 1993.

Wells, R.E. *What's Smaller Than a Pygmy Shrew?* Albert Whitman, 1995.

Ziefert, H. *The Magic Porridge Pot.* Puffin Books, 1997. [Alternate versions of story by dePaola & DeSpain.]

Versions of *Goldilocks & the Three Bears*

Aylesworth, J. *Goldilocks and the Three Bears*. Scholastic, 2003.

Barton, B. *The Three Bears.* Harpercollins, 1991.

Brett, J. *Goldilocks and the Three Bears*. G.P. Putnam's Sons, 1987.

Child, L. *Goldilocks and the Three Bears*. Disney Hyperion, 2008.

Marshall, J. *Goldilocks and the Three Bears*. Puffin 1988.

Ransom, C. *Goldilocks and the Three Bears*. School Specialty, 2005. [También en español: *Ricitos de Oro y los tres osos.*]

Rosales, M.B. *Leola and the Honeybears*. Scholastic, 1999.

Sanderson, R. *Goldilocks*. Little, Brown, 2009.

Spirin, G. *Goldilocks and the Three Bears*. Marshall Cavendish, 2009.