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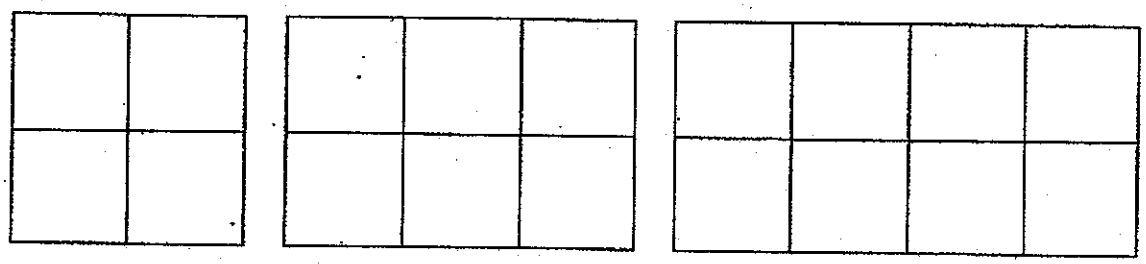
early mathematics education
INNOVATIONS

Summer Institute, Day 1
6/18/2012

Agenda

- *Begin with Breakfast*
- Investigation: Sharing Pizza
  - What is a whole? What is an equal part?
- Video Commentary
- CCSS for Mathematical Practice
  - What do they mean? How do they look & sound?
- Math Games: Mancala & 21
- *Break for Lunch*
- What do we know about how children learn math?
  - Big Ideas & Landscapes of Learning
- *Gathering to End the Day*

1. At Piece o' Pizza, the pieces in small, medium, and large pizzas are all the same size. A small has 4 pieces, a medium has 6 pieces, and a large has 8 pieces. You order a medium garlic-artichoke pizza and a large spinach-pineapple pizza. You eat $\frac{2}{6}$ of the garlic-artichoke and $\frac{3}{8}$ of the spinach-pineapple. How much of the pizza did you eat?



Small Medium Large

(Problem reprinted from *lenses on Learning, Module 2: Teacher Learning for Mathematics Instruction* by C.M.Grant et al. [Dale Seymour, 2003]).

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Practice #7: Look for and make use of structure.

Students:

- Look for and recognize mathematical significance
- Generalize relationships within and between problems (Math-to-Math connections)
- Apply a new idea to related problems

Teachers:

- Plan tasks and problems with patterns (e.g. number strings)
- Ask questions that focus students on the structure the problem
- Highlight different approaches for solving a problem

Practice #8: Look for and express regularity in repeated reasoning.

Students:

- Check work for sense, repeatedly.
- Notice patterns and connections that help them develop generalizations or "shortcuts"
- Explain what they are doing and why it makes sense.
- Explain why a generalization is true and useful.

Teachers:

- Ask about possible answers before, and reasonableness during and after computations.
- Use "think aloud" to model how to explain what they are doing and why it makes sense.
- Ask students to explain what they are doing and why it makes sense.
- Ask students if a generalization is always true.

HOW TO PLAY MANCALA

Mancala is an ancient family of board games, and there are numerous variants. This is a version of the basic game, known as two-rank Mancala and also known as Kalah.

What you need:

- Mancala board
- 36 markers/pieces (beans, stones, marbles)

What you do:

- The Mancala 'board' is made up of two rows of six holes, or pits, each. If you don't have a Mancala board handy, an empty egg carton is perfect.
- The board is placed between the two players with the long sides facing them; the six holes nearest each player belong to her/him. Each player has a larger 'store' to the right side of the Mancala board. (Cereal bowls work well for this purpose when using an egg carton for the board.)
- Three pieces – beans, marbles or stones -- are placed in each of the 12 holes. The color of the pieces is irrelevant.
- The game begins with one player picking up all of the pieces in any one of the holes on her/his side.
- Moving counter-clockwise, the player deposits one of the stones in each hole until the stones run out.
- If you run into your own store, deposit one piece in it. If you run into your opponent's store, skip it.
- If the last piece you drop is in your own store, you get a free turn.
- If the last piece you drop is in an empty hole on your side, you capture that piece and any pieces in the hole directly opposite.
- Always place all captured pieces in your store.
- The game ends when all six spaces on one side of the Mancala board are empty.
- The player who still has pieces on her/his side of the board when the game ends captures all of those pieces.
- Count all the pieces in each store. The winner is the player with the most pieces.

For variation:

- Play starting with 4 markers per hole (48 total).

How to Play 21

What you need

- standard deck of cards (no jokers) or any set of number cards

What you do:

- Someone must be the dealer for each hand in this game. Everyone else is a player. Keep going until each player has had a turn to be dealer.
- The dealer gives everyone two cards facedown.
- Each player looks at their two cards and adds their values to determine the total value of the hand. The value of a card is equal to the number on the card. All face cards are worth 10.
- Starting with the person to the left of the dealer, each player attempts to make the best possible hand by getting close to 21 points. A player with a point total far less than 21 may ask the dealer for another card by saying, "Hit." One card is then dealt to the player.
- Each player may continue to add cards until s/he thinks s/he has a good hand.
- If any added card sends the value of her/his hand higher than 21, the player has gone "bust" and s/he is no longer eligible to be a winner on that particular hand.
- If the player reaches a number that s/he is satisfied is close enough to 21, s/he says, "Stay," to instruct the dealer that s/he requires no more cards.
- The dealer goes around the group as many times as needed until all players have said "Stay."
- Each player who has not gone "bust" then announces her/his total and shows her/his cards. The player who is closest to 21 is the winner of that hand. There can be ties.

For variation:

- Play to a lower number, such as 17. (Take out the tens & face cards.)
- Aces can be worth either one or eleven, whichever value benefits the player.

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What do we know about how children learn about numbers & operations?

What does their *landscape of learning* look like?

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Big Ideas in Number & Operations

(1 of 3)

Topic	Big Ideas	Examples
Numerosity ☆☆☆☆☆ = 5	• Small collections can be intuitively perceived without counting (subitizing). • Quantity is an attribute of a set of objects.	• Children just "see" three objects and know it's 3. • 5 mice and 5 elephants are alike in quantity, though different in other ways.

Uses of Number
☆☆☆☆☆
5th
	• Numbers are used many ways, including:
	  • to indicate amount (cardinal)
	  • to specify position in a sequence (ordinal)
	  • to provide names for members of a set (nominal)
	  • to act as shared reference points (referential)
		• *Tommy has 5 books.* (cardinal)
		• *Ava is fifth in line today.* (ordinal)
		• Numbers on basketball jerseys, home addresses, telephone numbers (nominal)
		• *Let's meet at 5 pm on December 5.* (referential)

Counting
★★★★★
"1,2,3,4,5…5!"
	• Counting can be used to find out how many in a collection.
	• Counting has rules that apply to any collection.
	  • Counting words have to be said in the same order every time
	  • Each object in a set must be counted once and only once
	  • It does not matter in what order the objects within a set are counted
	  • The last number word produced is the amount of the entire set
		• *1, 2, 3, 4, 5…you have five stars!*
		• "One, four, two" doesn't give a correct answer
		• Children need strategies for keeping track, like touch-pointing or moving to another pile
		• Mixing up objects and counting again is a good exercise; the third object counted is not the only one that can "be" three
		• Being able to count is not the same as being able to answer "how many?"

Major support for the Early Mathematics Education Project is provided by the McCormick Foundation, CME Group Foundation, the U.S. Dept. of Education Investing in Innovation Fund (i3), Motorola Foundation, Chicago Public Schools Office of Early Childhood Education, Exelon Corporation, and the Robert and Isabelle Bass Foundation

Big Ideas in Number & Operations

(2 of 3)

Topic	Big Ideas	Examples
Place Value in Base Ten	•As numbers grow larger, we group by tens to create new units. •Because we group by tens, we can represent all numbers using ten digits (0 to 9), and there are patterns to how numbers are represented.	•Ten ones is one ten; ten tens is one hundred; ten hundreds is one thousand ... •"20, 21, 22, 23..." or "80, 90, 100, 110 ..."
	•The positions of digits in multi-digit numbers determine what unit they represent. •The digit "0" is important as a placeholder.	•Two tens and four ones describe the 24 single eggs we bought at the store. Four tens and two ones describe the 42 children in first grade. •"One Hundred & One Dalmatians" is a group of a hundred and one more. It is written as 101, representing one hundred-unit, zero ten-units and one one-unit.
	•The groups of ones, tens, hundreds (and so on) can be composed and de-composed in different ways.	•250 + 266 is simpler to compute when 256 is broken into 250 and 16. Then compute 250 + 250 (a double which many children know) and add on the extra 16 afterward.
Fractions	•Fractions are equal parts of a whole.	•Pizzas can be cut into 6 equal wedge-shaped pieces. •6 cars can be divided equally by giving 2 cars each to 3 children.
	•A whole or unit can be divided into equal parts in many different ways.	•A pizza can be divided into 4 or 6 or 8 equal slices. •6 cars can be divided equally into 2 groups of 3, 3 groups of 2, or 6 groups of 1.
	•A unit may be a single object or may be a collection of things.	•One whole pizza can be divided into equal slices. •In one group of 6 cars, 1/3 are red; 2 cars are red.

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Big Ideas in Number & Operations

(3 of 3)

Topic	Big Ideas	Examples
Changing Sets	• Sets can be *changed* by adding items (joining) or by taking some away (separating).	*•You have 2 balls and I have 3 balls. How many balls do we have altogether?* *•You had 60 cards, and you gave your friend 5. How many do you have now?*
Grouping & Partitioning	• One can quantify a collection by grouping items into equal sets.	*•Chris has 2 boxes of crayons with 4 in each box. How many crayons does Chris have altogether?* *•There are 20 children in the 2nd grade class. Sandy brings 40 cookies so each child can have two.* *•How many hands does it take to show 20 fingers?* *•How can 3 children share 9 toy cars fairly?*
Number Composition	•A quantity (whole) can be *decomposed* into equal or unequal parts; the parts can be *composed* to form the whole.	*•How many ways can you show 5 with fingers on both hands?* *• 100 can be 50 & 50 or 70 & 30 or 90 & 10.*
Comparing Sets ☆☆ < ☆☆☆	•Sets can be *compared* using the attribute of numerosity, and *ordered* by more than, less than and equal to.	*•I have a handful of raisins; Chris has a bowl-ful. Chris has more!* *•I have 1 pear and 1 peach; you have 2 apples. We have the same number of fruits.* *•Avery has 3 dirty plates, and Tracy has 4 dirty bowls. Who has fewer dishes to wash?* *•There are 6 fish and 3 snails in our aquarium. We have twice as many fish as snails.*
Solving Problems ?	• The four arithmetic operations (addition, subtraction, multiplication & division) are tools for solving problems about numbers. • In order to choose which operation to use, the solver must understand what is happening in the problem situation.	*•There is usually more than one way to solve the same problem. For example, subtraction or counting up are equally valid ways to find the difference between two numbers.* *•All word problems tell a story.*

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Teaching-Learning Strategies

- Turn & Talk
- Learners rephrase other learners' thinking
- Sharing multiple solutions or strategies without comment
- Teachers model students' thinking.
- Students explain or model their own thinking.

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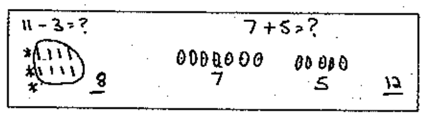
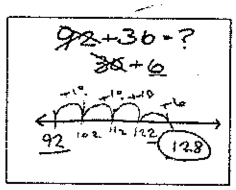
Modeling Students' Mathematical Thinking

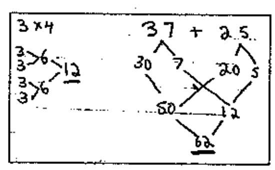
tallies or other marks



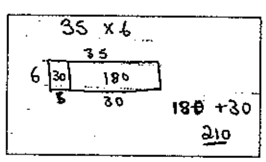
number line

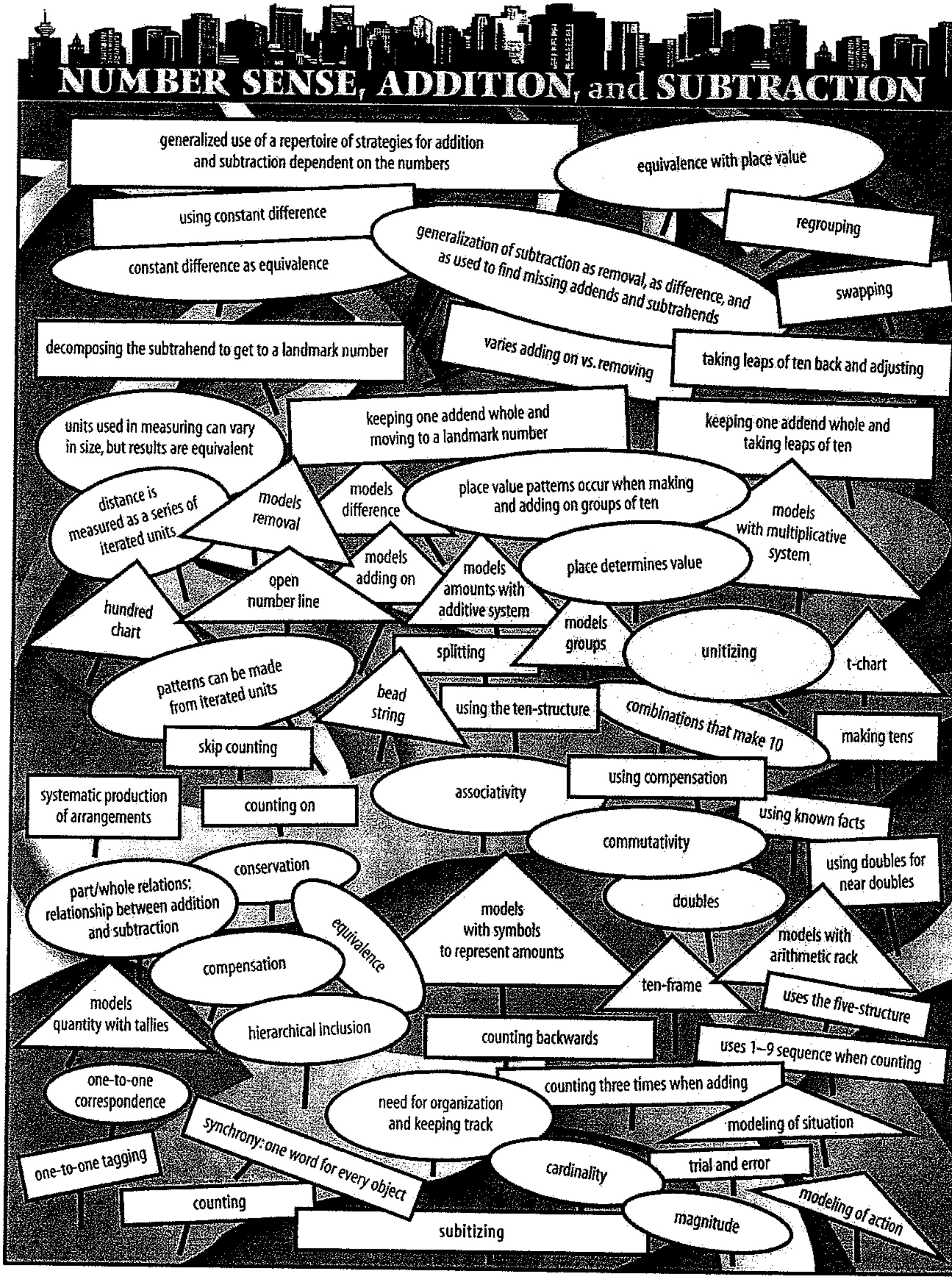


number tree



rectangle or array





The landscape of learning: number sense, addition, and subtraction on the horizon showing landmark strategies (rectangles), big ideas (ovals), and models (triangles).

From Fosnot & Dolk *Young Mathematicians at Work* series.
Printed at http://www.contextsforlearning.com/samples/k3LandscapeofLearning.pdf

Big Ideas, Concepts or Enduring Understandings in the Landscape of Learning for Addition & subtraction

concept	definition	Notes (examples, models, skills or other musings)
cardinality ***(principle of counting)***	When counting a set, the last number word produced names the amount of the entire set	 "1,2,3,4,5…5 stars!"
conservation of number	However a given quantity of items is arranged, the number stays the same.	

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Good Books for Teachers Thinking About Their Mathematical Practice

Andrews, A.G. & Trafton, P.R. *Little Kids – Powerful Problem Solvers: Math Stories from a Kindergarten Classroom.* Heinemann, 2002.

Burns, M. *About Teaching Mathematics, A K-8 Resource, 3rd ed.* MathSolutions, 2007.

Burns, M. *Math and Literature, Grades K-1.* MathSolutions, 2004.

Burns, M. & Sheffield, S. *Math and Literature, Grades 2-3.* MathSolutions, 2004.

Chapin, S.H., O'Connor, C. & Anderson, N.C. *Classroom Discussions: Using Math Talk to Help Students Learn, Grades K–6, 2nd Ed.* MathSolutions, 2009.

Diller, D. *Math Work Stations: Independent Learning You Can Count On, K-2.* Stenhouse, 2011.

Fosnot, C.T. & Dolk, M. *Young Mathematicians at Work: Constructing Fractions, Decimals and Percents.* Heinemann, 2002.

Fosnot, C.T. & Dolk, M. *Young Mathematicians at Work: Constructing Multiplication and Division.* Heinemann, 2001.

Fosnot, C.T. & Dolk, M. *Young Mathematicians at Work: Constructing Number Sense, Addition and Subtraction.* Heinemann, 2001.

Peterson, J. & Burns, M. *Math and Nonfiction, Grades K-2.* MathSolutions, 2004.

Richardson, K. *How Children Learn Number Concepts: A Guide to Critical Learning Phases!* Math Perspectives, 2012.

Richardson, K. *Developing Math Concepts in Pre-kindergarten.* Math Perspectives, 2008.

Russel, S.J., Schifter, D. & Bastable, V. *Connecting Arithmetic to Algebra: Strategies for Building Algebraic Thinking in the Elementary Grades.* Heinemann, 2011.

Siena, M. *From Reading to Math: How Best Practices in Literacy Can Make You a Better Math Teacher, Grades K–5.* MathSolutions, 2009.

Shumway, J. F. *Number Sense Routines: Building Numerical Literacy Every Day in Grades K-3.* Stenhouse, 2011.

Van de Walle, J.A. & Lovin, L.H. *Teaching Student-Centered Mathematics, Grades K-3.* Allyn and Bacon, 2006.

Wedekind, K.O. *Math Exchanges: Guiding Young Mathematicians in Small Group Meetings.* Stenhouse, 2011.

Whitin, D. & Wilde, S. *It's the Story that Counts.* Heinemann, 1995.

Wilburne, J.M., Keat,J.B. & Napoli, M. *Cowboys Count, Monkeys Measure, and Princesses Problem-Solve: Building Early Math Skills Through Story Books.* Brookes, 2011.

Wright, R.J., Stanger, G., Stafford, A.K. & Martland, J. *Teaching Number in the Classroom with 4-8 year olds.* Sage, 2006.