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INNOVATIONS

Learning Lab #6

Agenda

- Using What We Have Been Learning
- Investigation: Strategies for Subtraction
- Video Analysis: Kindergarten & 2nd Grade
  - Multiple Methods for Getting to the Answer
- Balanced Approach to Algorithms
- Lesson Analysis: Pre-k & 1st grade
  - Fostering understanding of solution strategies.
- High-Impact Strategies for Mathematics
  - Students explain or model their own thinking.
- High-Quality Books to Spark Mathematical Thinking & Action

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26 or 28 April 2012: Learning Lab 6

Content Focus: Using Number Sense & Algorithms to Solve Problems.
Strategy Focus: Students explain and model their own thinking.

Practical Plans for the Classroom: Activity Details to Remember	Mathematical Thinking: Ideas to Remember & Questions to Ask

Practical Plans for the Classroom: Activity Details to Remember	Mathematical Thinking: Ideas to Remember & Questions to Ask

Have fun with number sense, algorithms & all math!

Focus on "students explaining & modeling their own thinking" with your coach.

>>>> ***<u>The Spring Data Collection Period continues until June 8th</u>*** <<<<

We will see you again at the Summer Institute, Monday, June 18th through Thursday, June 21st

Reprinted from *Lenses on Learning, Module 1: Instructional Leadership in Mathematics* by Grant et al. (Dale Seymour, 2003),

Six Methods of Subtraction
Many Ways to Get to the Same Place

Working in small groups, you are going to explore several different methods for subtracting, or several "algorithms." Each method involves a systematic, step-by-step procedure. A few examples and a brief explanation are given for each method. For each of Methods A–F, please do the following on separate sheets of paper.

Step 1. Become familiar with each procedure by trying it out. Make up some more problems for yourself in order to develop facility with this approach.

Step 2. Discuss with others in the group why the method works. You may want to use words, manipulatives, diagrams, or any combination of these.

Method A

```
 4 1
  53
- 38
  15
```

You can't take 8 from 3, so you get a ten from the tens place. The 5 then becomes a 4, so you have 4 tens, and 13 in the ones place. Now you can subtract: 8 from 13 is 5, 3 from 4 is 1.

```
 3 9 1
  401
- 287
  114
```

You can't take 7 from 1, but you also can't get anything from the 0 tens, so you have to look in the hundreds place. Then it's basically the same as before.

Method B

```
  5 13
- 4 8   (38 → 48)
  15
```

You can't take 8 from 3, so you make the three 13. That means you have to make the 3 tens you're taking away into 4 tens. Then you subtract: 8 from 13 is 5, 4 from 5 is 1.

```
  6 10 13
- 4 10 5   (395)
  2  0 8
```

This is basically the same method as before, but you apply it repeatedly. The 9 tens become 10 tens, but those can be taken from 10 tens above.

Method C

$$\begin{array}{r} 53 \\ -38 \\ \hline 20 \end{array} \quad (-5)$$

3 minus 8 is negative 5. I'm going to hold that in my head while I do the tens.

50 minus 30 is 20.

Combine negative 5 and the 20 and my answer is 15.

If I wanted to, I could have written down the 20 and –5 below that and gotten 15.

$$\begin{array}{r} 401 \\ -287 \\ \hline 200 \\ -80 \\ \hline 120 \\ -6 \\ \hline 114 \end{array} \quad \begin{array}{l} (-80) \\ (-6) \end{array}$$

400 minus 200 is 200.

Nothing minus 80 is –80.

1 minus 7 is negative 6.

I take the 80 from the 200 and get 120.

Then I take away 6 more and get 114.

Method D

$$\begin{array}{r} 53 \\ -38 \\ \hline \end{array} \quad \begin{array}{r} 55 \\ -40 \\ \hline 15 \end{array}$$

53 – 38 is too hard, so add 2 to the 53 and 2 to the 38 making 55 – 40. The answer is 15.

$$\begin{array}{r} 401 \\ -287 \\ \hline \end{array} \quad \begin{array}{r} 404 \\ -290 \\ \hline \end{array} \quad \begin{array}{r} 414 \\ -300 \\ \hline 114 \end{array}$$

401 – 287 is too hard. Make it 404 – 290, but you're still not done. That becomes 414 – 300, which is easy, so subtract!

Method E

$$\begin{array}{r} 32{,}427 \\ -\,15{,}826 \\ \hline \end{array}$$

Start on the left. 3 – 1 is 2, but before I write that down, I look at the next column to see if there's going to be a problem. There is, so make the 2 a 1 and write it down.

I can't take 5 from 2, so I do a different procedure: figure out how many more it will take to make the 5 a ten (it's 5), then add that to the 2 to get 7. But before I write that down, I check the next column over.

There's a problem, so reduce the 7 by 1 to get 6 and write it down.

For 4 – 8, I need to do my alternative procedure: It'll take 2 more to make 10, then add that to 4 to get 6. But check the next column before going on.

I can't tell with 2 – 2, so check the next column. 7 – 6 won't be a problem, so write down the original 6 under 4 – 8. 2 – 2 is 0, and there is no problem in the next column so write it down.

7 – 6 is 1, and there is no next column, so write it down. The answer is 16,601.

Method F

$$\begin{array}{r} 53 \\ -\,38 \\ \hline \end{array} \qquad \begin{array}{r} 53 \\ +\,61 \\ \hline \not{1}14 \\ +\quad 1 \\ \hline 15 \end{array}$$

Transform into an addition problem by subtracting each number in the bottom (subtrahend) from 9. Thus, 3 becomes 6, 8 becomes 1. Now add. When done, drop the 1 in the largest place and add 1 to the ones place to get the answer of 15.

$$\begin{array}{r} 351 \\ -\,85 \\ \hline \end{array} \qquad \begin{array}{r} 351 \\ +\,914 \\ \hline \not{1}265 \\ +\quad 1 \\ \hline 266 \end{array}$$

Basically the same, but be sure to replace leading (invisible) zeros with 9.

Comparing the Methods

In your small groups, consider the following:

1. How are Methods A and B equivalent? Why do they result in the same answer? Discuss with others in the group why the methods work. You might use words, manipulatives, diagrams, or any combination of these.
2. If you have time, explain why two other methods result in the same answer.

In this chart, “algorithms” refers to paper-and-pencil procedures that were directly taught by the teacher or parent.

Table 2.3: Answers to 7 + 52 + 186 Given by Three Classes of Second Graders in May 1990

Algorithms $n = 17$	Some algorithms taught at home $n = 19$	No algorithms $n = 20$
9308		
1000		
989		
986		
938	989	
906	938	
838	810	
295	356	617
		255
		246
245 (12%)	245 (26%)	245 (45%)
		243
		236
		235
200	213	138
198	213	—
30	199	—
29	133	—
29	125	—
—	114	
—	—	
	—	
	—	
	—	

Note: Dashes indicate that the child declined to try to work the problem.

Kamii, C. (2004). *Young Children Continue to Reinvent Arithmetic, 2nd Grade* (2nd ed). Teachers College Press.

"Algorithms" is reprinted from the Everyday Mathematics *Teacher's Reference Manual: Early Childhood*, pages 56 - 59. For an expanded treatment of the subject, see the EM *Teacher's Reference Manual: Grades 1-3*, pages 95 - 115.

9.4 Algorithms

Computational algorithms are not a focus in *Pre-Kindergarten* and *Kindergarten Everyday Mathematics*. However, the question of whether and how algorithms should be taught in elementary school mathematics is a complex and controversial topic, so an overview of the *Everyday Mathematics* approach to algorithms is included in this section.

9.4.1 Algorithms and Procedures

As a teacher, you establish many procedures and routines to help your classroom run smoothly. For example, in the beginning of the year, you probably discuss the proper procedures for hanging up coats, lining up, and using materials. *Everyday Mathematics* encourages you to establish similar, but more mathematical, routines such as keeping a weather record or class calendar.

An *algorithm* is a well-defined, step-by-step procedure guaranteed to achieve a certain objective, often with several steps that "loop" as many times as necessary. For example, an algorithm for multiplication will produce the correct product no matter what the factors are.

A good algorithm is efficient, unambiguous, and reliable. Although you may be most familiar with the traditional elementary school procedures for adding, subtracting, multiplying, and dividing, there are many other algorithms both in mathematics and in real life. A computer program is an algorithm that specifies what a computer is to do at each step. The instructions for operating calculators or complicated equipment, such as FAX machines and VCRs, are forms of algorithms.

Formal algorithms are not introduced in Pre-Kindergarten or Kindergarten. In later grades, though, *Everyday Mathematics* includes a variety of both traditional computational algorithms and children's invented procedures. Inventing procedures is valuable because it:

- Promotes conceptual understanding and mental flexibility, both of which are essential for effective problem solving;
- Helps students learn about our base-ten place-value (decimal) system of numeration;
- Involves solving problems that the solver does not already know how to solve. Thus, asking children to devise their own computational methods provides valuable experience in solving nonroutine problems.

Traditional algorithms have advantages, too. They are generally efficient and can help children understand both the decimal number system and the underlying operations. Traditional algorithms also provide a common vocabulary for further development of mathematical ideas.

In addition to studying specific algorithms in *Everyday Mathematics*, children engage in activities to help them understand algorithms in a more general sense. Included are:

- Understanding specific algorithms or procedures provided by other people;
- Applying known algorithms to everyday problems;
- Developing algorithms and procedures when necessary;
- Realizing the limitations of algorithms and their procedures so that they are not used inappropriately;
- Adapting known algorithms to fit new situations.

Mathematics advances in part through the development of efficient procedures that reduce difficult tasks to routine exercises. An effective algorithm will solve an entire class of problems, thus increasing the user's mathematical power. The authors of *Everyday Mathematics*

NOTE: The term *algorithm* comes from the name *al-Khwarizmi*. Muhammad ibn Musa al-Khwarizmi (c. 780–850) was one of the greatest mathematicians of the Arab-Islamic world. We also have al-Khwarizmi to thank for the word *algebra*, which comes from *Hisab Aljabr w'al-muqabalah*, the title of one of his books.

have found that the study of paper-and-pencil computational algorithms at appropriate times can be valuable for developing algorithmic thinking in general.

9.4.2 Computational Algorithms: The *Everyday Mathematics* Approach

The *Everyday Mathematics* authors have been asked about the role of computational algorithms in elementary school mathematics. Before we address this issue, consider the following stories told by Professor Zalman Usiskin of the University of Chicago:

Scene 1: An Office Hal is preparing an end-of-the-month sales report. This involves doing many calculations, which he does, churning out each computation on paper. In walks the boss, horrified, saying, "Hal, why aren't you using a calculator? You're wasting valuable time!"

Scene 2: A Fourth-Grade Classroom The class is working on a page of difficult computational problems. Susie gets out her calculator and starts completing the assignment. The teacher walks over to Susie, horrified, saying, "Susie, put that calculator away or you'll get done too quickly!"

These two scenarios highlight the need to rethink the school mathematics curriculum in light of the widespread availability of calculators and computers outside of school. Children certainly still need:

- To know the meanings and uses of all the arithmetic operations in order to function in the practical world and to succeed in mathematics in high school and beyond;
- To know the basic addition and multiplication facts automatically, especially to help solve mental-arithmetic problems in our technological society;
- To understand and be able to apply paper-and-pencil algorithms for addition, subtraction, multiplication, and division of whole numbers, decimals, and fractions, especially in an environment of standardized testing.

Today's elementary school children also need to be prepared to be productive workers in the second half of the 21st century. Among other things, this means they need a conception of computation that takes into account advances in technology. For example, skill at judging the reasonableness of results is especially important for anyone using technology, whether sophisticated computer spreadsheets and modeling programs or simple four-function calculators. Estimation and approximation skills are also important both because many everyday applications of mathematics require quick, approximate answers and because one good way to judge whether a result is reasonable is to compare it to a sensible estimate. For all these purposes, mental arithmetic, both exact and approximate, is more useful than ever.

Along with increased attention to estimation and approximation, the broader approach to computation in *Everyday Mathematics* also includes paper-and-pencil algorithms taught with both efficiency and understandability in mind. That is, children are expected to know both *how* to add, subtract, multiply, and divide using paper and pencil methods and also *why* the methods they are using work.

Research carried out in the past 30 years by Kurt Van Lehn and others has shown that many children develop "buggy" algorithms that resemble standard procedures but do not work properly. In subtraction, for example, some children always subtract the smaller digit from the larger digit. Van Lehn has shown that bugs such as this develop because children are trying to carry out procedures they don't understand and can't remember well enough to reproduce accurately. Procedures that are well understood, on the other hand, are more easily recalled, are more easily "repaired" when they are not recalled accurately, and are more easily modified to fit new situations.

Children who solve mathematics problems using methods that they understand come to believe that mathematics is logical, that if they work at mathematics they can figure it out, and that doing mathematics can be enjoyable. Such beliefs are much more productive than those held by adults who believe that mathematics is a grab-bag of procedures that often don't make sense, that in mathematics more than other fields you either "get it" or you don't, and that the study of mathematics is often not much fun.

Because there are many paper-and-pencil methods that are both efficient and understandable, the authors of *Everyday Mathematics* believe that children should be exposed to paper-and-pencil algorithms for these reasons:

- Exploring different algorithms builds estimation skills and number sense and helps children see mathematics as a meaningful and creative subject.
- There are situations in which the most efficient or convenient way to carry out a computation is with paper and pencil.
- If taught properly for understanding, but without demands for "mastery" by all children by some fixed time, paper-and-pencil algorithms can reinforce children's understanding of our number system and of the operations themselves.

In the debate about algorithms, *Everyday Mathematics* takes a moderate position, combining elements from both the child-centered, invented-algorithms approach and the subject-matter-centered, traditional-algorithms approach. During the early phases of learning an operation, *Everyday Mathematics* encourages children to invent their own procedures. Children are asked to solve arithmetic problems from first principles about situations in which operations are used, before they develop or learn systematic procedures for solving such problems. This helps them to understand the operations better and also gives them valuable experience solving nonroutine problems.

Later, when children thoroughly understand the concept of the operation, several alternative algorithms are introduced. Some of these algorithms are based on approaches that many children devise on their own. Others are less likely to be discovered by children but have a variety of desirable characteristics. As children move through the grades, they are urged to experiment with various algorithms in order to become proficient at using at least one alternative.

NOTE: In *Mind Bugs: The Origins of Procedural Misconceptions*, cognitive scientist Kurt Van Lehn said this about using the traditional subtraction algorithm in some of his research:

> [O]rdinary multidigit subtraction . . . is a virtually meaningless procedure [for] most elementary school children. . . . When compared to procedures they use to operate vending machines or play games, subtraction is as dry, formal, and as disconnected from everyday interests as the nonsense syllables used in early psychological investigations were different from real words. This isolation is the bane of teachers. . . .

0•12 Fair Shares

Objective To introduce concrete equal-partitioning (fair sharing) experiences.

Key Mathematics Concepts and Skills

- Count objects. [Number and Numeration Goal 2]
- Compare quantities in two groups of objects. [Number and Numeration Goal 4]
- Explore part-whole relationships to divide a quantity into equal groups. [Operations and Computation Goal 1]

Other Skills Cooperation, Pretending and Role Play

Terms to Use fair, equal, half, same

Materials pairs of teddy bears or other toy animals; plastic cups; miniature plastic food or other small manipulatives; small paper plates (optional); paper food pieces (optional)

Planning Tip Put an even number of manipulatives (between 2 and 10) into several plastic cups. Each cup should have a different number of objects. You need at least one cup for each child or pair of children.

Main Activity

☐ Whole Group ☑ Small Group ☑ Partners ☑ Center

Give each child or pair of children two teddy bears. Tell the children that their bears are hungry. You may want to give children a plate for each bear. Hand out the cups containing an even number of manipulatives. Explain that inside the cups are pieces of pretend food for the bears. To be fair, each bear should get the same number of pieces.

NOTE Children may find feeding the bears very appealing. Allow them time to play with the bears and food before asking them to make fair shares.

Observe as children distribute the food to their bears. Do they divide the pile visually and then count how many are in each pile? Do they divide the objects by distributing them one at a time (one for this bear, one for that bear, and so on)? Do they use trial and error? Regardless of their strategies, are they able to divide the set of objects equally? Once children have distributed the food, have them count to see whether both bears have the same number of pieces. Ask: *Is it fair? Do both bears have the same amount of food?* Allow children to collect the food, switch cups with each other, and feed their bears with a different number of pieces.

Once children are familiar with the activity, you can leave the bears and cups of food out in the Math Center for children to use. Remind children to count their shares to make sure they are fair, or they can ask a friend to check.

Adjusting for Age and Development

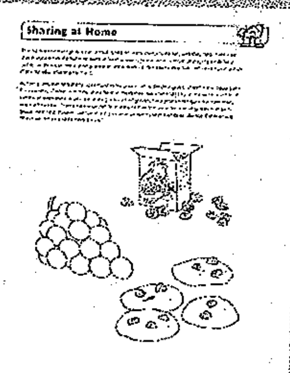
Give children cups with an odd number of pieces to introduce the idea of sharing with some left over. You may want to use paper food pieces, so you can cut the remainder in half. Observe what children do with the extra piece of food. Children may offer many solutions, including setting it aside or giving it to one bear who is "hungrier." If nobody suggests it, ask whether it would be fair to cut the extra piece in half. Cut the remaining piece in half and ask children to share the halves fairly. They can check by counting the number of whole pieces and then the number of half pieces for each bear. For another challenge, children can try sharing pieces among more than two bears.

Connections

Snack Connection There are many opportunities for equal partitioning at snack and meal times. Ask children to help you figure out how much food each child should get and how to share equally. Listen for children's use of words such as *fair, same, equal,* and *half* and note whether they seem to understand their mathematical meanings.

Dramatic Play Connection Place the toys and food in the Dramatic Play Center. Encourage children to continue sharing the food as part of their dramatic play.

Family Connection You may want to use the Sharing at Home Family Connection (*Math Masters,* page 90) to encourage families to find opportunities to divide groups of items into fair shares.



Math Masters, p. 90

EM 1st grade

5•5 Animal Weights

Objective To introduce addition of 2-digit numbers.

1 Teaching the Lesson

Key Activities

Children use base-10 blocks to find the total weight of pairs of animals.

Key Concepts and Skills

- Use base-10 blocks to model 2- and 3-digit whole numbers. [Number and Numeration Goal 3]
- Exchange base-10 longs and cubes to show different representations of the same number. [Number and Numeration Goal 3]
- Use base-10 blocks to find sums of 2- and 3-digit numbers. [Operations and Computation Goal 2]
- Model parts-and-total diagrams for addition number stories. [Operations and Computation Goal 4]

Ongoing Assessment: Informing Instruction See page 381.

materials

- ☐ *Math Journal 1*, Activity Sheets 7 and 8
- ☐ Home Link 5•4
- ☐ Teaching Masters (*Math Masters*, pp. 132–137)
- ☐ Tens-and-Ones Mat (*Math Journal 1*, p. 81)
- ☐ domino for overhead projector (optional)
- ☐ slate
- ☐ base-10 blocks (longs and cubes)

See Advance Preparation

2 Ongoing Learning & Practice

Children play *Shaker Addition Top-It* to practice addition facts.

Children practice and maintain skills through Math Boxes and Home Link activities.

Ongoing Assessment: Recognizing Student Achievement Use journal page 88. [Measurement and Reference Frames Goal 1]

materials

- ☐ *Math Journal 1*, p. 88
- ☐ Home Link Master (*Math Masters*, p. 138)
- ☐ Game Master (*Math Masters*, p. 353)
- ☐ two 6-sided dice, or two polyhedral dice marked 0–9
- ☐ 20 pennies or counters per group

3 Differentiation Options

READINESS

Children solve parts-and-total problems using base-10 blocks.

ENRICHMENT

Children play *Animal Weight Top-It* to explore comparing quantities, addition of 2-digit numbers, and finding differences.

materials

- ☐ Teaching Master (*Math Masters*, p. 139)
- ☐ Animal Cards (*Math Journal 1*, Activity Sheets 7 and 8)
- ☐ paper plates
- ☐ base-10 blocks (cubes and longs)

See Advance Preparation

Additional Information

Advance Preparation For Part 1, *Math Masters*, pages 132–137, provide a set of Animal Cards for problem solving. There are 2 animals per master for a total of 12 animals. Activity Sheets 7 and 8 provide each child with a set of Animal Cards that match those on the masters. For the optional Readiness activity in Part 3, divide plates into parts-and-total diagrams as shown on page 383.

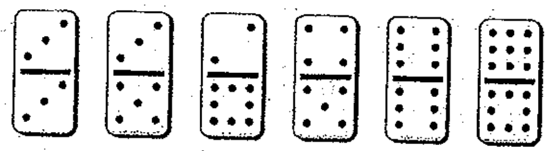
Technology

Assessment Management System
Math Boxes, Problem 3
See the **ITLG.**

Getting Started

Mental Math and Reflexes

Show dominos, such as the following, on the overhead projector or draw them on the board.



On their slates, have children write a number fact for the numbers on each of the dominoes.

Math Message

A fox weighs 14 pounds. A cat weighs 7 pounds. What is their total weight?

Home Link 5·4 Follow-Up

Briefly go over the problems and record the answers on the board in both cents and dollars-and-cents notation.

1 Teaching the Lesson

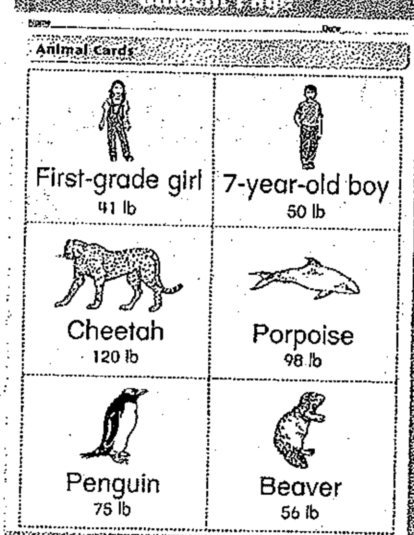
▶ Math Message Follow-Up

WHOLE-CLASS DISCUSSION

Children share their solution strategies, which may include modeling with counters or with base-10 blocks or counting up on the number line or number grid. Show children ways to record their various strategies. To summarize, draw a parts-and-total diagram and write a number model on the board. Call attention to the abbreviation *lb* for *pound.* Tell children to look for the word *pound* in their math lesson today. To support English language learners, give them an object to hold that weighs about one pound. Circulate the object in the class so that others have an opportunity to hold it.

Total	
21	
Part	Part
14	7

14 lb + 7 lb = 21 lb



Math Journal 1, Activity Sheet 7 (front)

▶ Using Base-10 Blocks to Find Total Weight

WHOLE-CLASS ACTIVITY

(*Math Journal 1,* Activity Sheets 7 and 8)

Have children look through the set of Animal Cards from Activity Sheets 7 and 8. Point out that the same animal is shown on both sides of a card. A weight for the animal is given on one side and a height or length is given on the other side.

Links to the Future

This is the first lesson in which children use base-10 blocks for solving problems and number stories involving the addition of 1- and 2-digit whole numbers. Using base-10 blocks for solving number stories involving the addition and subtraction of two 2-digit numbers is a Grade 2 Goal.

Explain that the weights and heights shown on the cards are measures one might expect such an animal to have. However, weights and heights vary from animal to animal, just as different children weigh different amounts and are different heights.

Write a unit box for pounds on the board. Display your pictures of the raccoon and the fox from *Math Masters,* pages 134 and 135. Ask children to take out their base-10 blocks.

Demonstrate on the overhead projector or on the board how to use base-10 blocks to find the total weight of the raccoon (23 lb) and the fox (14 lb). On the Tens-and-Ones Mat (*Math Journal 1,* page 81), place 2 longs and 3 cubes to represent the weight of the raccoon and 1 long and 4 cubes to represent the weight of the fox. Together, there are 3 longs and 7 cubes, which represent the number 37 (pounds), the total weight of the two animals.

Display your pictures of the cat (7 lb) and the koala (19 lb) from *Math Masters,* pages 132 and 133. On their Tens-and-Ones Mats, children use their base-10 blocks to find the total weight of these animals. Ask them to describe what they did to find the answer. As a first step, they probably got a total of 1 long and 16 cubes (1 long and 9 cubes for the koala and 7 cubes for the cat). Review how to exchange the 16 cubes for 1 long and 6 cubes, for a total of 2 longs and 6 cubes, or 26.

Repeat this routine for the following pairs of animals. Each time, children use their base-10 blocks to represent the weight of the animals.

▷ The boy and the girl 5 longs + 4 longs, 1 cube = 9 longs, 1 cube; or 91

▷ The raccoon and the eagle 2 longs, 3 cubes + 1 long, 5 cubes = 3 longs, 8 cubes; or 38

▷ The cat and the eagle 7 cubes + 1 long, 5 cubes = 1 long, 12 cubes = 2 longs, 2 cubes; or 22

▷ The koala and the fox 1 long, 9 cubes + 1 long, 4 cubes = 2 longs, 13 cubes = 3 longs, 3 cubes; or 33

Allow a few children to come to the board and demonstrate adding 2-digit animal weights. Encourage children to use vocabulary such as *cubes, longs,* and *pounds.*

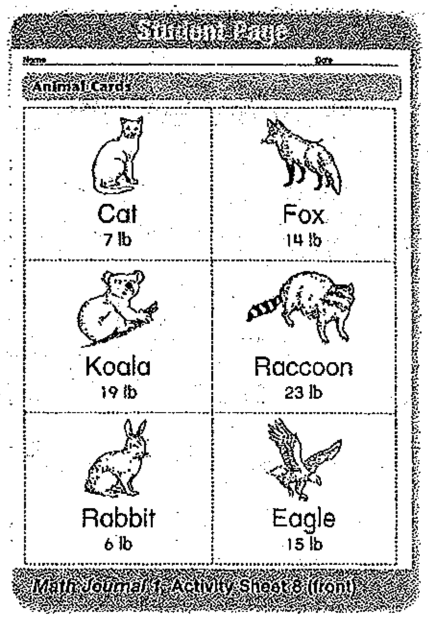
Ongoing Assessment: Informing Instruction

Watch for children who do not remember how to exchange 10 cubes for 1 long. Prompt the exchange by showing them that a line of 10 cubes is the same length as 1 long.

Adjusting the Activity

Have children find the total weight of the penguin and the girl. 7 longs, 5 cubes + 4 longs, 1 cube = 11 longs, 6 cubes = 1 flat, 1 long, 6 cubes = 116 You might also ask children to find the total weight of three animals.

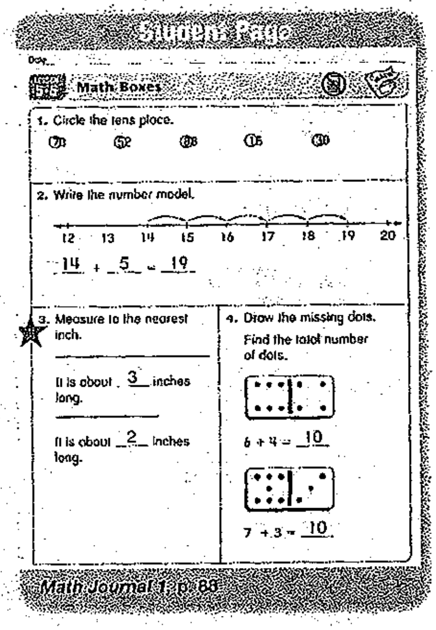
AUDITORY • KINESTHETIC • TACTILE • VISUAL



NOTE The *Math Masters* pages picture the following:

Cheetah: 120 lb	Fox: 14 lb
Koala: 19 lb	Rabbit: 6 lb
Beaver: 56 lb	First-grade girl: 41 lb
Raccoon: 23 lb	7-year-old boy: 50 lb
Penguin: 75 lb	Eagle: 15 lb
Cat: 7 lb	Porpoise: 98 lb

Science Link Invite volunteers to tell any facts they know about the animals shown on the masters. Encourage children to name the type of covering each animal has (fur, feathers, or skin), as well as its method of movement (flying, walking/running, hopping, or swimming).


Student Page

Math Boxes

1. Circle the tens place.

2. Write the number model.

12 13 14 15 16 17 18 19 20

14 + 5 = 19

3. Measure to the nearest inch.

It is about 3 inches long.

It is about 2 inches long.

4. Draw the missing dots. Find the total number of dots.

6 + 4 = 10

7 + 3 = 10

Math Journal 1, p. 88

2 Ongoing Learning & Practice

▶ Playing *Shaker Addition Top-It*

PARTNER ACTIVITY

(*Math Masters*, p. 353)

Children practice addition facts by playing *Shaker Addition Top-It*. For detailed instructions, see Lesson 4-12.

▶ Math Boxes 5•5

INDEPENDENT ACTIVITY

(*Math Journal 1*, p. 88)

Mixed Practice Math Boxes in this lesson are paired with Math Boxes in Lesson 5-7. The skills in Problem 4 preview Unit 6 content.

Ongoing Assessment: Recognizing Student Achievement

Math Boxes Problem 3 ★

Ask children to draw a picture of one object in their classroom that is shorter than both of the line segments on **Math Boxes, Problem 3**. Use children's drawings to assess their ability to compare lengths. Children are making adequate progress if the object they choose to draw is shorter than 2 inches.

[Measurement and Reference Frames Goal 1]

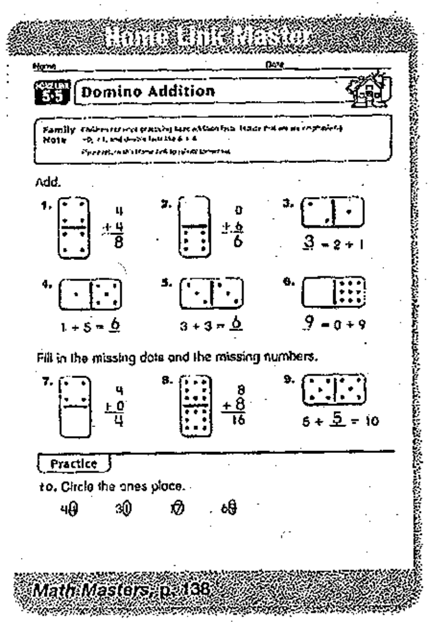
Writing/Reasoning Have children draw, write, or verbalize an answer to the following question: *How do you know how many tens are in a number?* A reasonable answer should reference the digit in the tens place.

▶ Home Link 5•5

INDEPENDENT ACTIVITY

(*Math Masters*, p. 138)

Home Connection Children find missing addends and sums for dominoes.


Home Link Master

Domino Addition

Add.

1. 4 + 4 = 8
2. 0 + 6 = 6
3. 3 = 2 + 1
4. 1 + 5 = 6
5. 3 + 3 = 6
6. 9 = 0 + 9

Fill in the missing dots and the missing numbers.

7. 4 + 0 = 4
8. 8 + 8 = 16
9. 5 + 5 = 10

Practice

10. Circle the ones place.

40 30 70 60

Math Masters, p. 138

3 Differentiation Options

READINESS

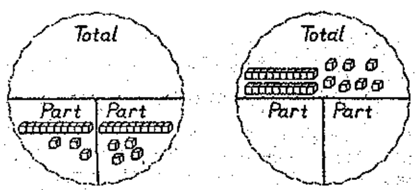
INDEPENDENT ACTIVITY

5–15 Min

▶ Solving Parts-and-Total Problems

(*Math Masters,* p. 139)

To explore solving parts-and-total problems using a concrete model, have children model number stories on a plate divided into three sections. For each number story on *Math Masters,* page 139, children put base-10 blocks in each of the Part sections of the plate, and then move the Parts into the Total section to solve the problems. Model the example for children. Allow children to work independently to complete the remaining problems.



Parts-and-total diagram plates

ENRICHMENT

PARTNER ACTIVITY

15–30 Min

▶ Playing *Animal Weight Top-It*

(*Math Journal 1,* Activity Sheets 7 and 8)

To further explore comparing quantities, adding of 2-digit numbers, and finding differences, have children play a variation of *Top-It.*

Directions

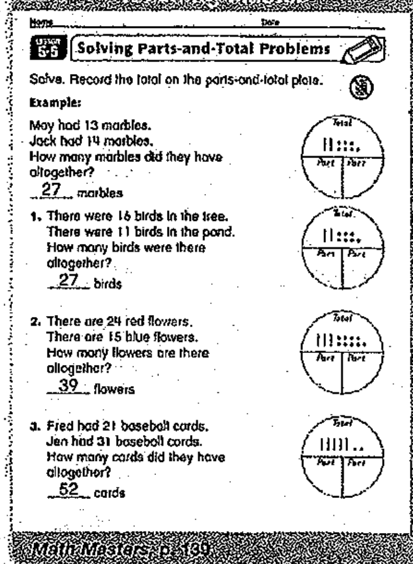
To begin, players combine their decks of Animal Cards, mix them, and place them in a stack on a table, weight-side down.

1. Player A turns over the two top cards from the stack. Player B turns over the next card from the top of the stack.
2. Player A finds the total weight of the animals on his or her two cards and decides whether these two animals together weigh more or less than Player B's animal.
3. If they weigh more, Player A scores the difference between his or her total and Player B's card. If they weigh less, Player B scores the difference between his or her card and Player A's total. In case of a tie, neither player scores.
4. Players then trade roles.

The game is over when all cards from the stack have been played.

NOTE Children can use base-10 blocks and/or their number grids on the inside back covers of their journals to find the answers.

Teaching Master



Name ______ Date ______

Lesson 5·5 **Solving Parts-and-Total Problems**

Solve. Record the total on the parts-and-total plate.

Example:

May had 13 marbles.
Jack had 14 marbles.
How many marbles did they have altogether?
27 marbles

1. There were 16 birds in the tree.
There were 11 birds in the pond.
How many birds were there altogether?
27 birds

2. There are 24 red flowers.
There are 15 blue flowers.
How many flowers are there altogether?
39 flowers

3. Fred had 21 baseball cards.
Jen had 31 baseball cards.
How many cards did they have altogether?
52 cards

Math Masters, p. 139

erikson
early mathematics education
INNOVATIONS

High Quality Books to Spark Children's Thinking About Number & Operations

Anno, M. *Anno's Counting Book.* HarperCollins Publishing, 1975.

Baker, K. *Quack and Count.* Voyager Books, 1999.

Bang, M. *Ten, Nine, Eight.* Greenwillow Books, 1983.
También en español: Bang, M. *Diez, Nueve, Ocho.* Mulberry.

Brisson, P. *Benny's Pennies.* Dragonfly Books, 1993.

Burns, M. *Spaghetti and Meatballs for All!* Scholastic, 1997.

Carle, E. *1, 2, 3 to the Zoo.* PaperStar, 1968/1987.

Crews, D. *Ten Black Dots.* Greenwillow Books, 1986.
También en español: Crews, D. *Los Diez Puntos Negros.* Greenwillow Books.

Curtis, M. *Six Empty Pockets.* Children's Press, 1997.

Cuyler, M. *100th Day Worries.* Scholastic, 2000.

Dee, R. *Two Ways to count to Ten: A Liberian Folktale.* Henry Holt, 1988.

Demi. *One grain of rice: A mathematical folktale.* Scholastic, 1997.

Ehlert, L. *Fish Eyes.* Sandpiper, 1990.

Falwell, C. *Feast for 10.* Scholastic, 1993.

Fleming, D. *Count!* Henry Holt, 1992.

Fox, M. *Night Noises.* Harcourt, 1989.

Giganti, P. *Each orange had 8 slices: A counting book.* Greenwillow, 1992.
Giganti, P. *How many snails? A counting book.* Greenwillow, 1988.

Guy, G.F. *Fiesta!* [In English & en español] Scholastic, 1996.

Hamm, D.J. *How Many Feet in the Bed?* Aladdin, 1991.

Harris, T. *Splitting the Herd.* Millbrook, 2008.

Harshman, M. *Only One.* Scholastic, 1993.
Harshman, M. *Only One Neighborhood.* Scholastic, 2007.

Hoban, T. *26 letter and 99 cents.* Greenwillow Books, 1987.
Hoban, T. *Count and See.* Simon & Schuster, 1972
Hoban, T. *Let's Count.* Greenwillow Books, 1999.
Hoban, T. *More, Fewer, Less.* Greenwillow Books, 1998.

Hong, L.T. *Two of everything.* Albert Whitman & Company, 1993.

Hopkins, L.B. & Barbour, K. *Marvelous Math: A Book of Poems.* Aladdin, 1997.

Hutchins, P. *The Doorbell Rang.* Mulberry, 1986.
También en español: Hutchins, P. *Llaman a la Puerta.* Mulberry.

Jonas, A. *Splash!* Greenwillow Books, 1995.

Kellogg, S. *How Much is a Million?* Harper Collins Publishing, 1985.

Lewis, J.P. *Arithmetickle*. Harcourt, 2002.

Mahy, M. *17 Kings and 42 Elephants*. Dial Books, 1987.

McKissack, P.C. *A Million Fish ... More or Less*. Scholastic, 1992.

McMillan, B. *Eating fractions*. Scholastic, 1991.
También en español: McMillan, B. *A Comer Fractiones!* Scholastic.

Merriam, E. *12 Ways to Get to 11*. Aladdin Paperbacks, 1993.

Murphy, F. *Ben Franklin and the Magic Squares*. Random House, 2001.

Neuschwander, C. *Amanda Bean's Amazing Dream*. Scholastic, 1998.

Nolan, H. *How much, how many, how far, how heavy, how long, how tall is 1000?* Scholastic, 1995.

Ochiltree, D. *Cats Add Up!* Scholastic, 1998.

Pallotta, J. *Apple Fractions*. Scholastic, 2002.
Pallotta, J. *Count to a Million*. Scholastic, 2003.
Pallotta, J. *One Hundred Ways to Get to 100*. Scholastic, 2003.

Pilegard, V.W. *The Warlord's Beads*. Pelican, 2001.

Princzes, E. *One Hundred Hungry Ants*. Houghton Mifflin Company,1993.
Princzes, E. *A Remeinder of One*. Houghton Mifflin Company,1995.

Rocklin, J. *One Hungry Cat*. Scholastic, 1997.

Sayre, A.P. & Sayre, J. *One is a Snail, Ten is a Crab*. Candlewick, 2003.

Schlein, M. *More than one*. Scholastic,1996.

Simon, C. *One Happy Classroom*. Scholastic, 1997.

Slater, T. *...98, 99, 100! Ready or Not, Here I Come!* Scholastic, 1999.

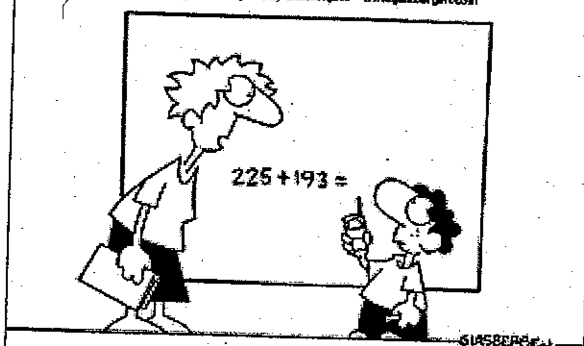
Sturges, P. *Ten Flashing Fireflies*. NorthSouth, 1995.

Tang, G. *The Best of Times*. Scholastic, 2002. (ages 7-12)
Tang, G. *The Grapes of Math*. Scholastic, 2001. (ages 7-12)
También en español: Tang,G. *Come Una y Cuenta 20*. Everest
Tang, G. *Math Appeal: Mind-stretching Math Riddles.*. Scholastic, 2003. (ages 7-12)
Tang, G. *Math Fables: Lessons that Count*. Scholastic, 2004. (ages 3- 6)
Tang, G. *Math for All Seasons*. Scholastic, 2002. (ages 5-8)
También en español: Tang, G. *Un, Dos, Tres, El Ano se Fue*. Everest.
Tang, G. *Math Potatoes: Mind-stretching Brain Food*. Scholastic, 2005. (ages 8-13)
Tang, G. *Math-terpieces: The Art of Problem-solving*. Scholastic, 2003. (ages 5-12)

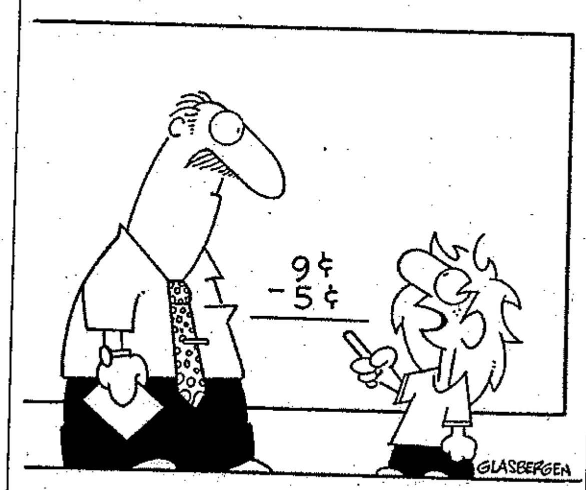
Walsh, E.S. *Mouse Count*. Harcourt, 1991
También en español: Walsh, E.S. *Cuenta Ratones*. Fondo de Cultura Economica.

Wise, W. *Ten Sly Piranhas: A Counting Story in Reverse*. Puffin, 1993.

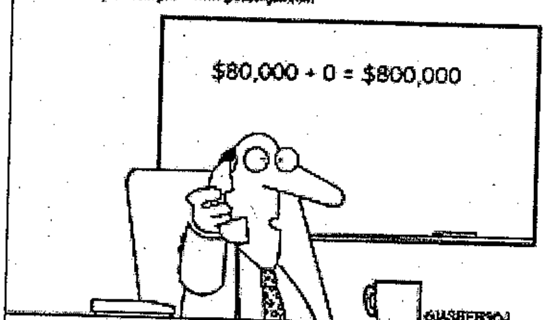
Math Humor from Randy Glasbergen



"You have to solve this problem by yourself. You can't call tech support."



"If the brightest minds on Wall Street can't figure out the economy, then what do you expect from *me?*"



"That's right, I've decided to give myself zero pay raise this year."