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early mathematics education
INNOVATIONS

Learning Lab #4

# Afternoon Handouts
# 2nd

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***Who lives here?***

Grandpa is a funny guy. He says that at his house there are 14 feet and 2 tails. Who might live at Grandpa's house?

Show all your work. Explain in words how you found your answer. Tell why you took the steps you did to solve the problem.

Make sure you
— show all your work in solving the problem,
— clearly label your answer,
— write in words how you solved the problem,
— write in words why you took the steps you did to solve the problem, and
— write as clearly as you can.

**Exemplars Math K-2 Sample (www.exemplars.com)**

***Placas de Automóvil / License Plates***

***Espanol***

En un paseo reciente buscamos placas de automóviles que tuviereran 3 números. Demuestra todas las placas que encontramos con números que al sumarse equivalen al número 6.

Explica todo tu trabajo utilizando dibujos, números, y palabras.

***English***

On a recent car trip we looked for license plates that had 3 numerals on them. Show all of the license plates that we found that had numbers that added up to 6.

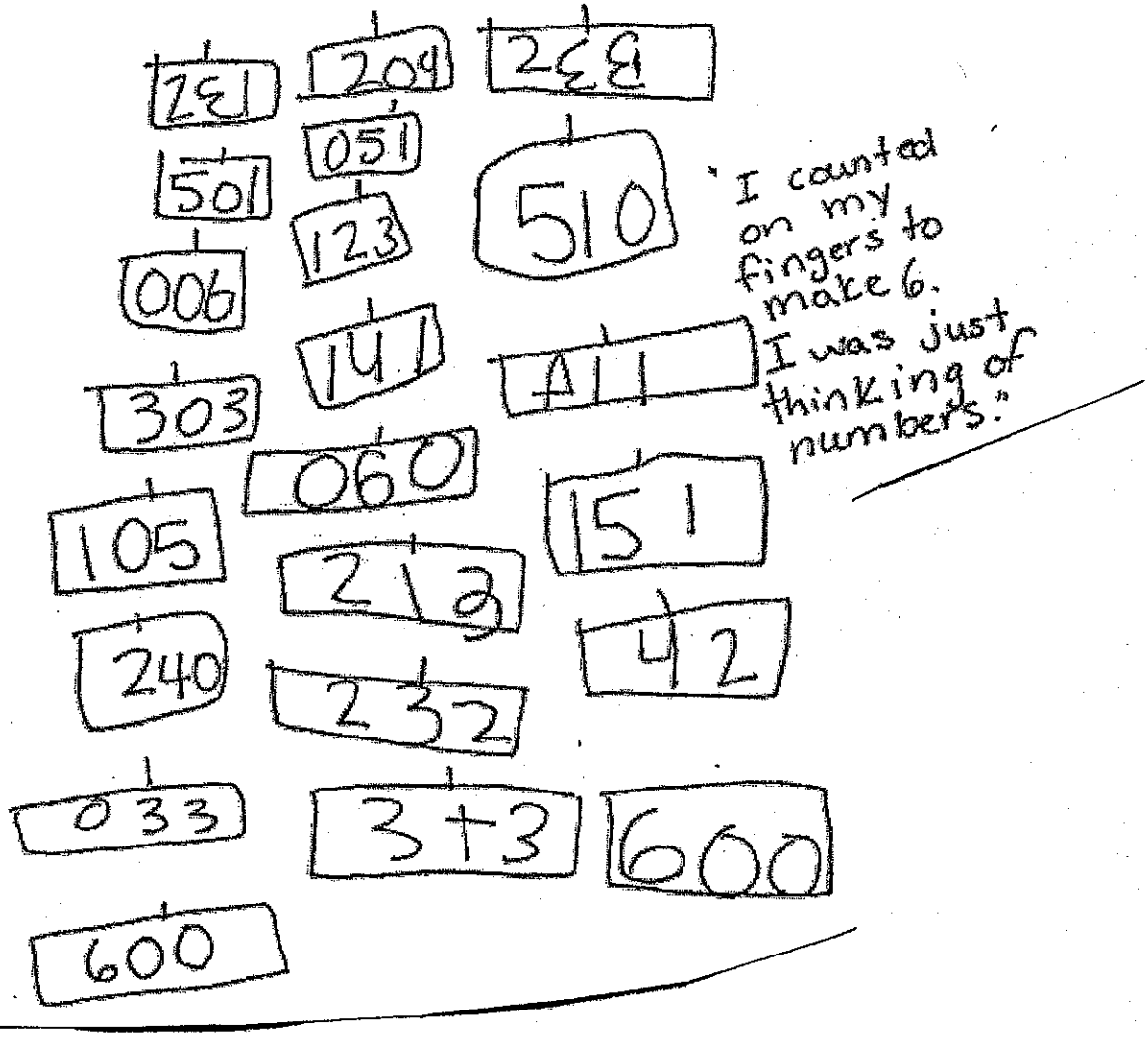
Explain all of your work using pictures, numbers, and words.

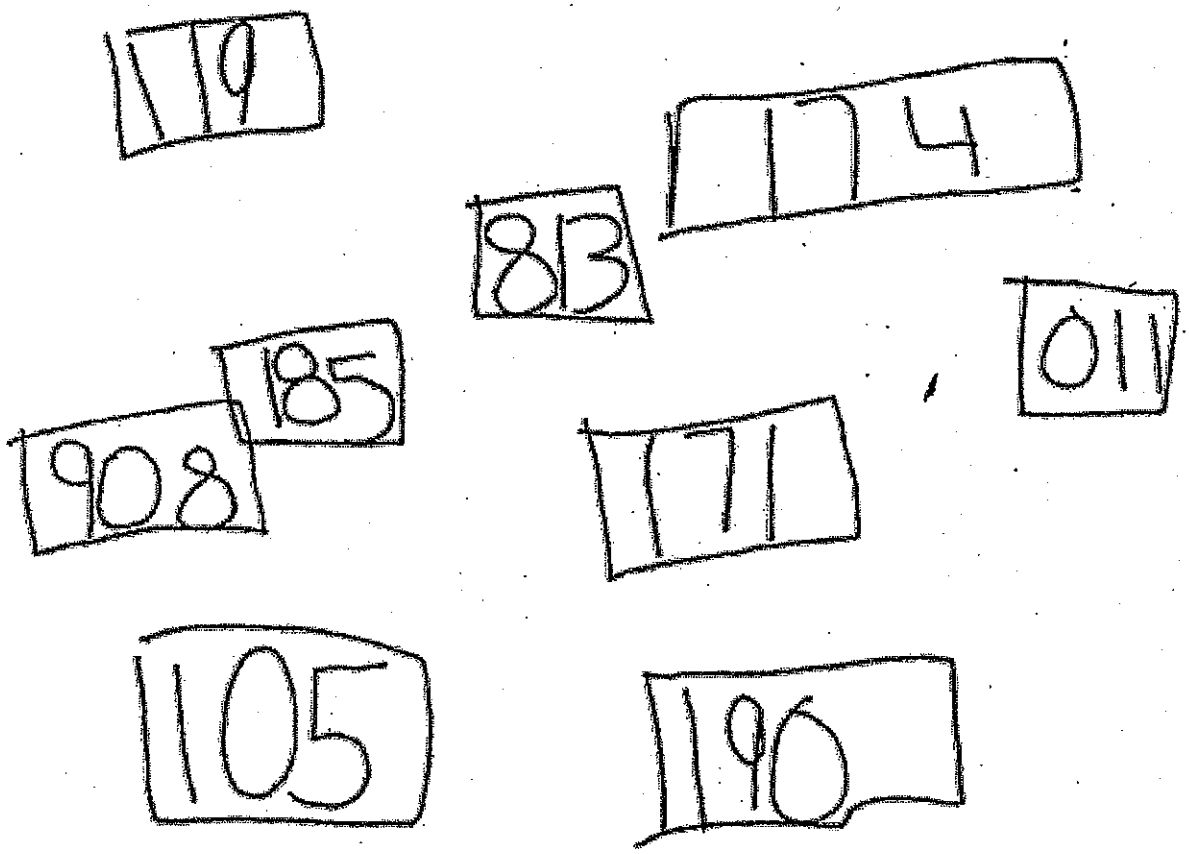
---

You have four samples of student solutions to this problem.

- Examine them and discuss them with others at your table.
- Put them in order of mathematical complexity – novice, apprentice, practitioner, expert.
- What evidence of student thinking can you see that helps you assign the samples to different levels of complexity?

| *Level* | *What you notice about student work sample* |
|---|---|
| **Novice** | |
| **Apprentice** | |
| **Practitioner** | |
| **Expert** | |


251
209
258
051
501
510
123
006
141
411
303
060
105
151
213
240
42
232
033
3+3
600
600
I counted on my fingers to make 6. I was just thinking of numbers."




I poot homrs on plates.

$\boxed{222}$ trip/s 2+2+2=6

$\boxed{114}$ dobl #s 1+1+4=6

$\boxed{141}$ Swich arond- 1+4+1=6

$\boxed{411}$ 4+1+1=6

$\boxed{006}$ dobl 0's 0+0+6=6

$\boxed{060}$ Swich arond

$\boxed{600}$

$\boxed{330}$ dobl 3's 0+3+3=6

$\boxed{303}$ Swich

$\boxed{033}$

The 0 can be any plase.

$\boxed{123}$ 1+2+3=6

$\boxed{321}$ → backwrds

$\boxed{231}$ 3+2+1=6

$\boxed{132}$ $\boxed{312}$ $\boxed{213}$

$\boxed{510}$ 5+1+0=6

$\boxed{015}$ If ther are 3 numbrs then ther are 6 paterns

$\boxed{150}$

$\boxed{051}$

$\boxed{501}$

$\boxed{105}$

1+0+5=6

$\boxed{420}$

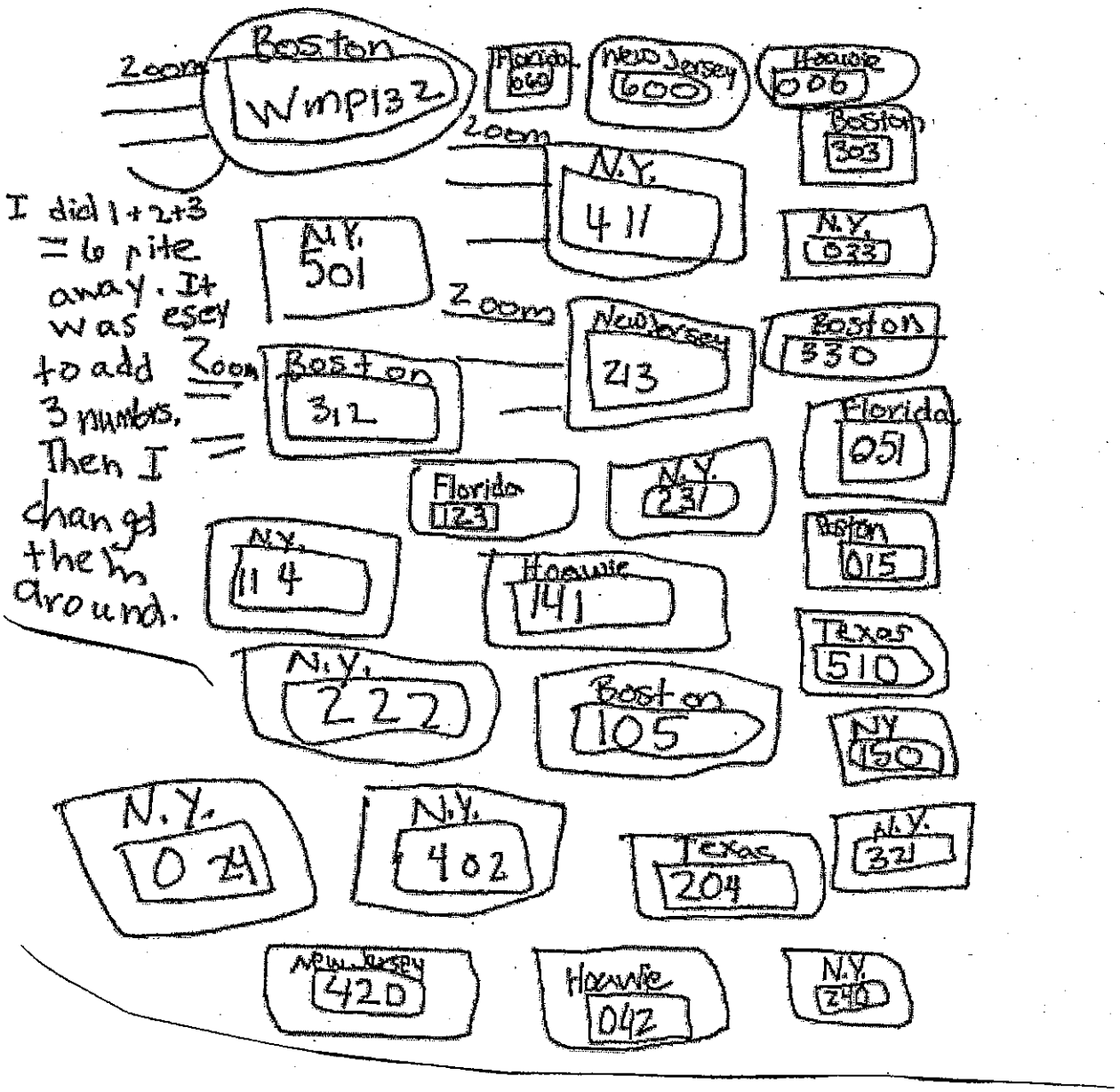
$\boxed{024}$ 4+2+0=6

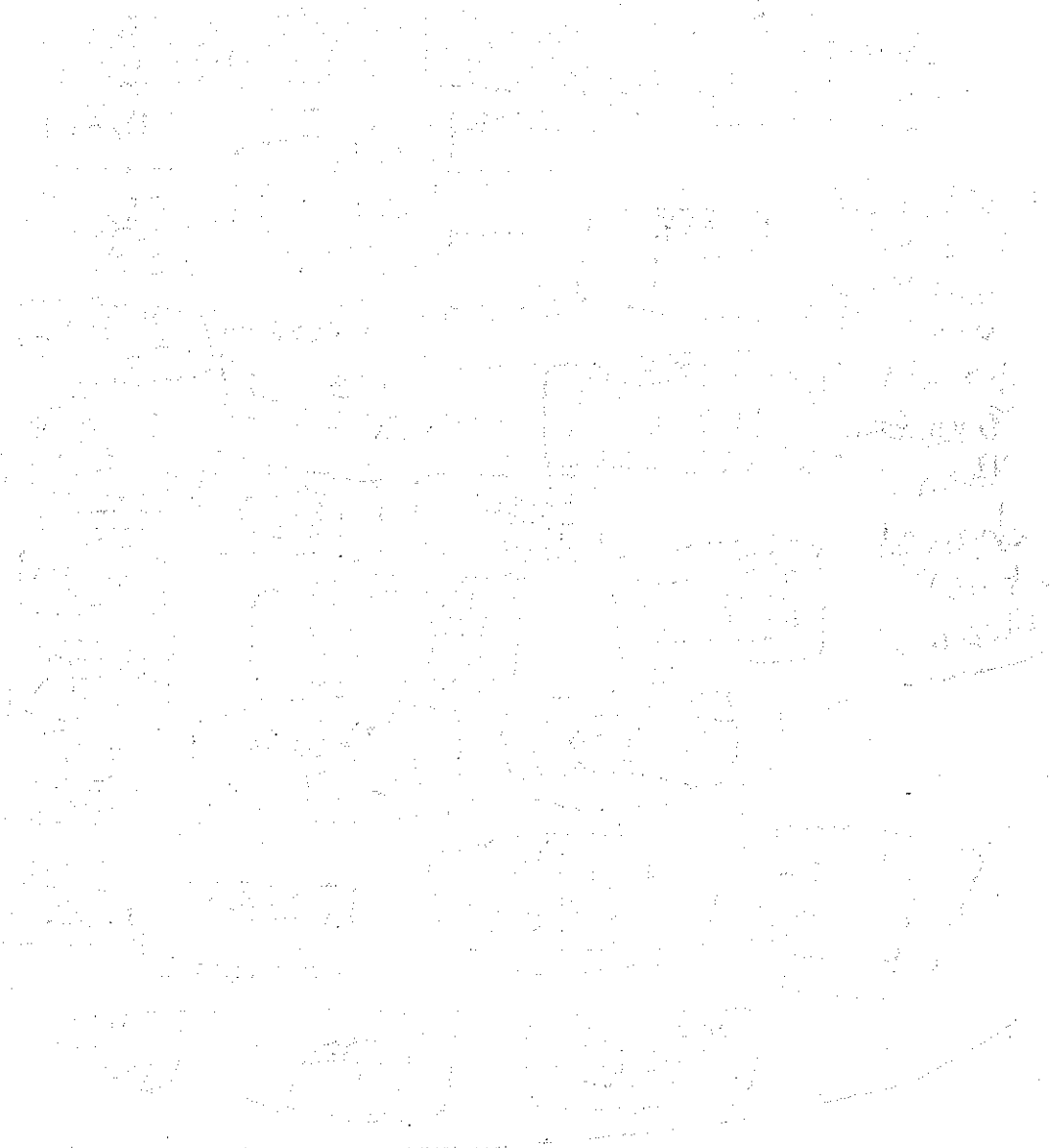
$\boxed{240}$

$\boxed{042}$ The same

$\boxed{204}$

$\boxed{402}$


Boston
Wmp132
Zoom
Florida
060
New Jersey
600
Hawaii
006
Boston
303
Zoom
N.Y.
411
N.Y.
033
N.Y.
501
Zoom
New Jersey
213
Boston
330
Zoom
Boston
312
Florida
051
Florida
123
N.Y.
231
Boston
015
N.Y.
114
Hawaii
141
Texas
510
N.Y.
222
Boston
105
NY
150
N.Y.
024
N.Y.
402
Texas
204
N.Y.
321
New Jersey
420
Hawaii
042
N.Y.
240
I did 1+2+3 = 6 rite away. It was esey to add 3 numbs. Then I changed them around.



# 6•4 Mixed Addition and Subtraction Stories

**Objective** To guide children in selecting and completing an appropriate diagram to help solve an addition or subtraction problem.

## 1 Teaching the Lesson

### Key Activities

Children categorize number stories as change, parts-and-total, or comparison problems; fill in an appropriate diagram to record known and missing information; and write number models to summarize their solutions.

### Key Concepts and Skills

- Explain solution strategies. [Operations and Computation Goal 2]
- Solve 2-digit addition and subtraction problems within number stories. [Operations and Computation Goal 2]
- Identify change, parts-and-total, and comparison number stories. [Operations and Computation Goal 4]

**Ongoing Assessment: Informing Instruction** See page 398.

**Ongoing Assessment: Recognizing Student Achievement** Use journal page 141. [Operations and Computation Goal 2]

### materials

- *Math Journal 1*, p. 141
- Home Link 6•3
- Teaching Aid Master (*Math Masters*, p. 437); 2 copies per child
- Transparency (*Math Masters*, p. 437; optional)
- slate
- number grid; number line (optional)

*See* **Advance Preparation**

## 2 Ongoing Learning & Practice

Children practice ballpark estimation.

Children practice and maintain skills through Math Boxes and Home Link activities.

### materials

- *Math Journal 1*, pp. 142 and 143
- Home Link Masters (*Math Masters*, pp. 167 and 168)
- calculator

## 3 Differentiation Options

### materials

**READINESS**

Children use counters to solve a number story told by a partner.

- Teaching Aid Masters (*Math Masters*, pp. 419 and 437)
- *Minute Math®+*, p. 16
- counters or tool-kit coins

**ENRICHMENT**

Children make up parts-and-total, comparison, and change number stories.

**EXTRA PRACTICE**

Children make up addition number stories.

## Additional Information

**Advance Preparation** For the second activity in Part 1, you might want to make an overhead transparency of *Math Masters*, page 437.

## Technology

**Assessment Management System**
Journal page 141, Problems 1 and 2
See the iTLG.

# Getting Started

## Mental Math and Reflexes

Write multiple-addend problems on the board. Encourage children to look for combinations that will make the addition easier.

- ○○○ ? = 43 + 5 + 7 55
- ○○○ 6 + 8 + 9 = ? 23; no particularly easy way to add these numbers
- ○○○ ? = 1 + 15 + 29 + 5 50

## Math Message

*Make a list of some things you like to collect.*

## Home Link 6-3 Follow-Up

Ask comparison questions about the survey data such as the following:

- How many more people like watermelon than like grapes?

Ask parts-and-total questions, such as the following:

- What is the total number of people who like apples and pears?

# 1 Teaching the Lesson

## ▶ Math Message Follow-Up

INDEPENDENT ACTIVITY

(*Math Journal 1*, p. 141)

Ask children to describe some of the things they like to collect. Record their responses on the board. Sample answers: coins, shells, sports cards, dolls, stuffed animals

Have each child select two items to use as topics for number stories on the journal page. Different children might select different things.

Problems 1 and 2 on the journal page have answer blanks within their number stories. Ask children to select one item for each problem and to write that item in all the empty boxes. For example, suppose "coins" is one of the selected items. Problem 1 would be completed as follows: Colin has 20 coins. Fiona has 30 coins. How many coins do they have in all?

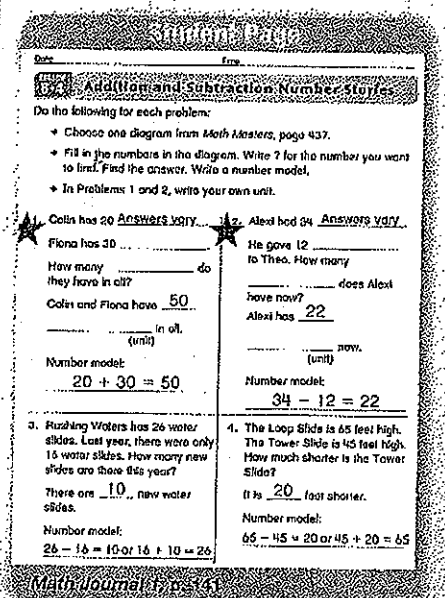
## ▶ Selecting Diagrams and Solving Number Stories

INDEPENDENT ACTIVITY

(*Math Journal 1*, p. 141; *Math Masters*, p. 437)

Until now, lessons have focused on one type of number story at a time. For example, all the problems in Lesson 6-2 were based on comparison stories, and the comparison diagram was the only diagram used. In this lesson, children are asked to categorize addition and subtraction stories: They must decide which type of story (change, parts-and-total, or comparison) best matches the problem at hand and then use the appropriate diagram to find a solution.

The journal page contains four number stories, and the master has two sets of diagrams. Each set includes a change diagram, a parts-and-total diagram, and a comparison diagram.



Student Page

Date _____ Time _____

LESSON 6-4 Addition and Subtraction Number Stories

Do the following for each problem:

- Choose one diagram from *Math Masters*, page 437.
- Fill in the numbers in the diagram. Write ? for the number you want to find. Find the answer. Write a number model.
- In Problems 1 and 2, write your own unit.

1. Colin has 20 Answers vary.
Fiona has 30 _____
How many _____ do they have in all?
Colin and Fiona have 50 _____ (unit) in all.
Number model:
20 + 30 = 50

2. Alexi had 34 Answers vary.
He gave 12 _____ to Theo. How many _____ does Alexi have now?
Alexi has 22 _____ (unit) now.
Number model:
34 − 12 = 22

3. Rushing Waters has 26 water slides. Last year, there were only 16 water slides. How many new slides are there this year?
There are 10 new water slides.
Number model:
26 − 16 = 10 or 16 + 10 = 26

4. The Loop Slide is 65 feet high. The Tower Slide is 45 feet high. How much shorter is the Tower Slide?
It is 20 feet shorter.
Number model:
65 − 45 = 20 or 45 + 20 = 65

Math Journal 1, p. 141

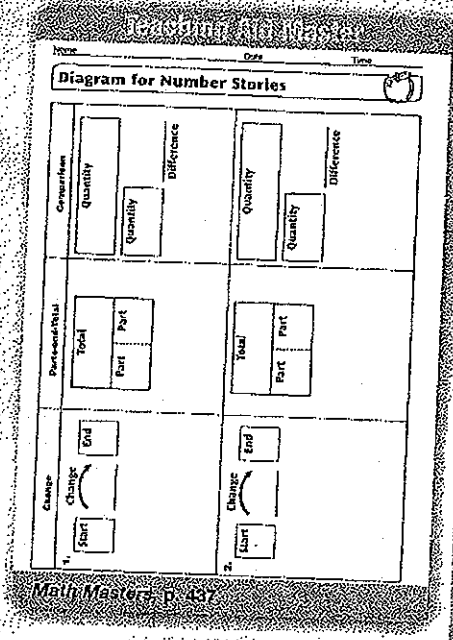

Name Date Time

Diagram for Number Stories

Comparison: Quantity, Quantity, Difference; Quantity, Quantity, Difference

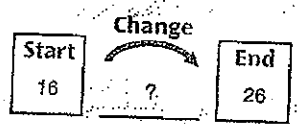
Part-and-Total: Total, Part, Part; Total, Part, Part

Change: End, Change, Start; End, Change, Start

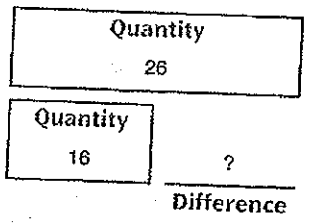
1. 2.

Math Masters, p. 437

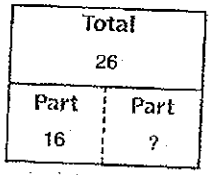
**NOTE** Do not expect children to always select the same diagram. Different children might think of a problem in different ways and select the diagrams that match their thinking.




Change Diagram




Comparison Diagram




Parts-and-total Diagram

## Ongoing Assessment: Recognizing Student Achievement

Journal page 141
Problems 1 and 2 ★

Use journal page 141, Problems 1 and 2 to assess children's progress solving number stories. Children are making adequate progress if they can solve the problem using the number grid, number line, or any other manipulative. Some children may be able to solve the problem using mental math and then write a number model that summarizes the problem.

[Operations and Computation Goal 2]

## Ongoing Assessment: Informing Instruction

Watch for children who are having difficulty choosing a diagram. Ask them first to explain how they view the problem. Then direct them toward the diagram that best matches their way of thinking. Alternatively, pick a diagram and ask children if they are able to explain how to solve the problem using that diagram.

For each number story, children do the following:

1. Choose one diagram they think is appropriate.
2. Fill in the diagram by writing known numbers in the appropriate places and by writing a question mark to represent the unknown number.
3. Calculate the sum or difference and solve the problem.
4. Write a number model to summarize the problem.

Solve Problems 3 and 4 with the class. Display an overhead transparency of *Math Masters*, page 437 or draw the three kinds of diagrams on the board. Ask a volunteer to select one of the diagrams, explain his or her choice, and model the solution at the board. When the problem has been solved in one way, ask if anyone selected one of the other diagrams and solved it in a different way.

Do not force any number story into a particular mold or say that there is a best diagram for the problem. As the following examples show, there may be several ways to view a problem—and to select a diagram:

**Example 1:** *Problem 3 viewed as a change problem*

*Think:* Last year there were 16 slides (the Start number). New slides were added (the Change number). Now, this year, there are 26 slides (the End number). I want to find the Change number.

**Example 2:** *Problem 3 viewed as a comparison problem*

*Think:* I'm comparing the number of slides this year (the larger Quantity, 26) to the number of slides last year (the smaller Quantity, 16). I want to find how many more slides there are this year (the Difference).

**Example 3:** *Problem 3 viewed as a parts-and-total problem*

*Think:* I know there are 26 slides in all (the Total). 16 of them (the first Part) were there last year, and some new slides (the second Part) were added this year. I want to find the second Part.

*Summary:* The purpose of diagrams is to help children organize the information in a number story, identify the missing information, and determine whether to add or subtract to solve the problem. Children should be encouraged to select the diagram that best matches the way they see the problem. There is no right or wrong diagram for any given problem; what matters is that the chosen diagram matches the child's thinking and is used as a tool for finding the correct answer.

## 2 Ongoing Learning & Practice

### ▶ Reviewing Ballpark Estimation

INDEPENDENT ACTIVITY

(*Math Journal 1*, p. 142)

Children complete this journal page by finding a ballpark estimate for each problem. They then solve the problem using a calculator. When most children have completed this journal page, have them compare their estimates to the exact answer to see if the exact answer is in the ballpark of their estimate. Discuss.

### ▶ Math Boxes 6•4

INDEPENDENT ACTIVITY

(*Math Journal 1*, p. 143)

**Mixed Practice** Math Boxes in this lesson are paired with Math Boxes in Lesson 6-2. The skill in Problem 6 previews Unit 7 content.

### ▶ Home Link 6•4

INDEPENDENT ACTIVITY

(*Math Masters*, pp. 167 and 168)

**Home Connection** Children make up number stories to match diagrams. Then they solve their number stories and complete a number model.

## 3 Differentiation Options

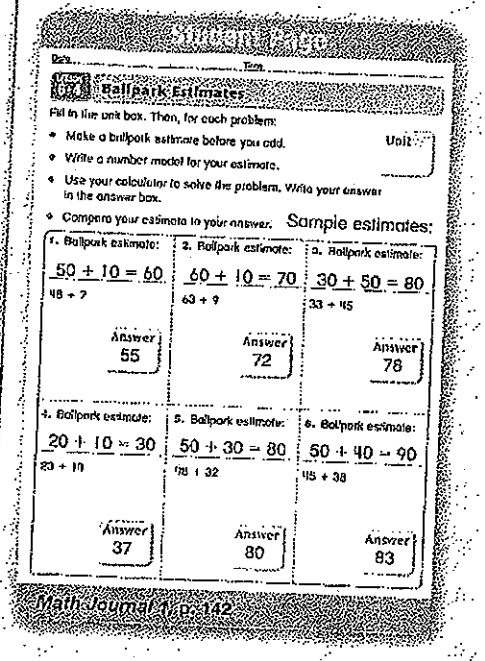
READINESS

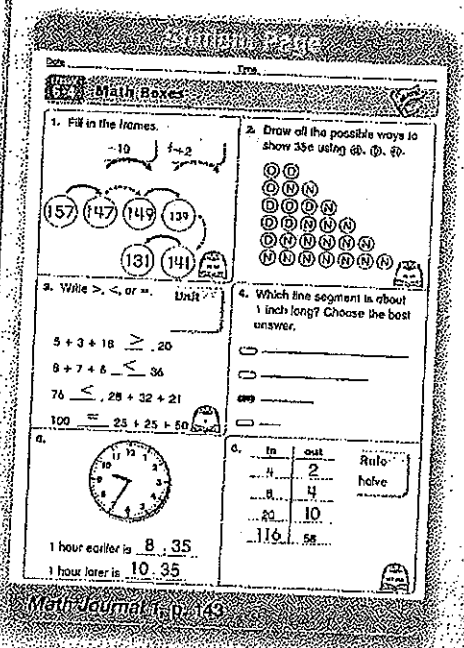
### ▶ Solving a Partner's Number Story

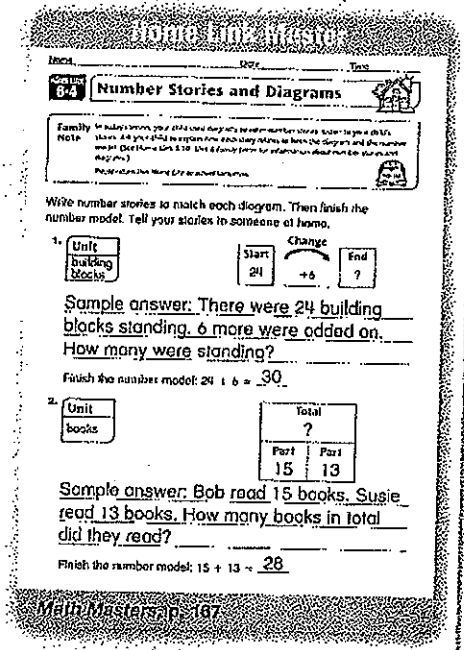
PARTNER ACTIVITY

5–15 Min

To provide experience with number stories, have children tell a partner a number story. Encourage them to tell change stories and parts-and-total stories. Children can use counters or coins to model their stories. They use the laminated diagrams and act out





Name _____ Date _____ Time _____

6-4 Number Stories and Diagrams

Family Note

Write number stories to match each diagram. Then finish the number model. Tell your stories to someone at home.

1. Unit: building blocks

Start: 24 Change: +6 End: ?

Sample answer: There were 24 building blocks standing. 6 more were added on. How many were standing?

Finish the number model: 24 + 6 = 30

2. Unit: books

Total: ? Part: 15 Part: 13

Sample answer: Bob read 15 books. Susie read 13 books. How many books in total did they read?

Finish the number model: 15 + 13 = 28

Math Masters, p. 167

the story with counters and coins. For example, *Julia had 48 cents, and Marcus had 35 cents. If Marcus gives his money to Julia, how much money will she have?* Children can place the coins in a change diagram and figure out the total. *How much did Julia and Marcus have all together?* Children can place the coins in the parts-and-total diagram and figure out the total.

ENRICHMENT

INDEPENDENT ACTIVITY

15–30 Min

## Using Different Diagrams to Write Number Stories

(*Math Masters*, pp. 437 and 419)

To further explore addition and subtraction number stories, have children solve a number story using all three diagrams. Tell children a number story. For example say: *Francis wanted to buy a toy turtle from the store. He had 67 cents. The turtle cost 83 cents.* Have them fill in all three diagrams (with a question mark for the missing number) to show how all three diagrams could be used to solve the problem correctly. When they have finished, have them explain their thinking.

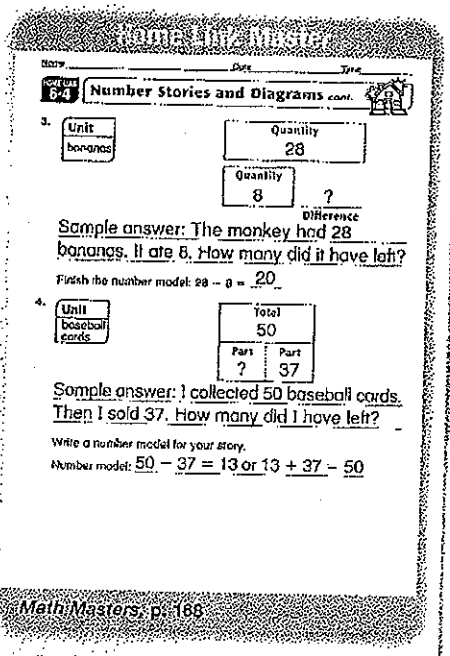
EXTRA PRACTICE

SMALL-GROUP ACTIVITY

5–15 Min

## *Minute Math+*

To offer children more experience with making up number stories, see the following page in *Minute Math+*: p. 16.



Name _____ Date _____ Time _____

6-5 Number Stories and Diagrams cont.

3. Unit: bananas

Quantity: 28

Quantity: 8 Difference: ?

Sample answer: The monkey had 28 bananas. It ate 8. How many did it have left?

Finish the number model: 28 − 8 = 20

4. Unit: baseball cards

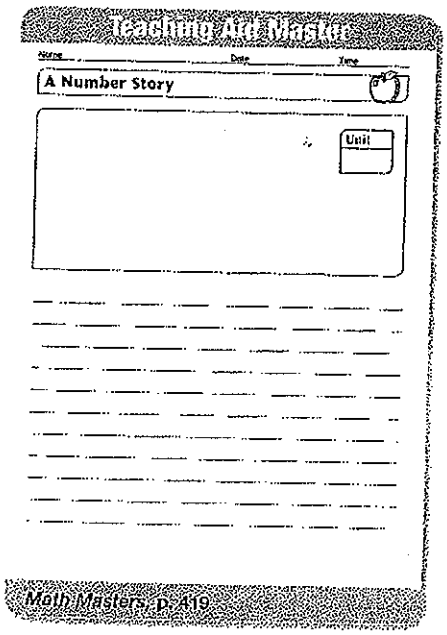
Total: 50 Part: ? Part: 37

Sample answer: I collected 50 baseball cards. Then I sold 37. How many did I have left?

Write a number model for your story.

Number model: 50 − 37 = 13 or 13 + 37 = 50

Math Masters, p. 166



Name _____ Date _____ Time _____

A Number Story

Unit

Math Masters, p. 419

Lesson 6

# Snack Shop Addition and Subtraction

## Lesson Overview

*Estimated Class Sessions*: 1

Students use calculators to solve addition and subtraction problems based on the *Shooting Star Snack Shop Children's Menu* introduced in Unit 9. Students calculate total snack shop bills and determine the amount of change or the additional money needed.

## Key Content

- Solving problems involving money.
- Solving addition and subtraction problems using calculators.
- Estimating sums and differences.

## Math Facts

DPP item Q is a quiz on the subtraction facts for Group A.

## Homework

Assign the *Snack Shop Carryout 2* Homework Pages to give students more practice adding and subtracting priced items. Children need to take home the menu to complete this assignment.

## Assessment

1. Use Assessment Indicator A4 and the *Observational Assessment Record* to document students' abilities to solve problems involving money.
2. For assessment, ask students to solve a subtraction problem two different ways.
3. Use DPP item Q, Assessment Indicator A6, and the *Observational Assessment Record* to document students' fluency with the subtraction facts for Group A.
4. Transfer appropriate documentation from the Unit 11 *Observational Assessment Record* to students' *Individual Assessment Record Sheets.*

# Materials List

## Supplies and Copies

| Student | Teacher |
|---|---|
| **Supplies for Each Student Pair**<br>• calculator<br>• counters such as connecting cubes<br>• base-ten pieces | **Supplies**<br>• overhead calculator, optional |
| **Copies**<br>• 1 copy of *200 Chart* per student (*Unit Resource Guide* Page 41) | **Copies/Transparencies**<br>• 1 transparency of *Shooting Star Snack Shop Children's Menu*, optional (*Student Guide* Page 305)<br>• 1 transparency of *Snack Shop Sample* (*Unit Resource Guide* Page 81) |

*All blackline masters including assessment, transparency, and DPP masters are also on the Teacher Resource CD.*

## Student Books

*Shooting Star Snack Shop Children's Menu* (*Student Guide* Page 305)
*Snack Shop Bills 2* (*Student Guide* Pages 306–308)
*Snack Shop Carryout 2* (*Student Guide* Pages 309–310)

## Daily Practice and Problems

DPP items Q–R (*Unit Resource Guide* Pages 25–26)

Note: Classrooms whose pacing differs significantly from the suggested pacing of the units should use the Math Facts Calendar in Section 4 of the *Facts Resource Guide* to ensure students receive the complete math facts program.

## Assessment Tools

*Observational Assessment Record* (*Unit Resource Guide* Pages 11–12)
*Individual Assessment Record Sheet* (*Teacher Implementation Guide,* Assessment section)

# Daily Practice and Problems

Suggestions for using the DPPs are on pages 78–79.

**Q. Subtraction Facts Quiz: Group A** (URG p. 25)

A. $1 - 0 =$ ___

B. $2 - 1 =$ ___

C. $6 - 2 =$ ___

D. ___ $= 5 - 3$

E. $4 - 2 =$ ___

F. ___ $= 3 - 1$

G. $1 - 1 =$ ___

H. $3 - 2 =$ ___

I. $6 - 4 =$ ___

J. $2 - 0 =$ ___

K. ___ $= 4 - 3$

L. $2 - 2 =$ ___

M. $4 - 1 =$ ___

N. $5 - 2 =$ ___

Explain how you solved Question N.

**R. Story Problems** (URG p. 26)

Make up a story for each of these number sentences.

A. $45 - 7 =$ ___

B. $71 +$ ___ $= 89$

C. $13 -$ ___ $= 7$

Name _____ Date _____

**Shooting Star Snack Shop**
**Children's Menu**

| Food | | | |
|---|---|---|---|
| Pizza Slice | 79¢ | | |
| Taco | 59¢ | | |
| Grilled Cheese Sandwich | 89¢ | | |
| Turkey Sandwich | 99¢ | | |
| Peanut Butter and Crackers | 49¢ | | |
| Bagel | 29¢ | | |
| Potato Chips | 25¢ | | |
| Pretzels | 25¢ | | |
| Brownie | 35¢ | | |
| Fruit Salad Cup | 65¢ | | |
| Carrot Sticks | 29¢ | | |
| Chicken Noodle Soup | 55¢ | | |
| Chili | 75¢ | | |
| **Drinks** | **Small** | **Medium** | **Large** |
| Milk | 25¢ | 40¢ | 55¢ |
| Lemonade | 99¢ | 55¢ | 79¢ |
| Orange Juice | 55¢ | 70¢ | 85¢ |
| Hot Chocolate | 69¢ | 75¢ | 99¢ |

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*Student Guide* - page 305

## Teaching the Activity

Have children turn to the *Shooting Star Snack Shop Children's Menu* Activity Page as you display the *Snack Shop Sample* Transparency Master. Read the problem and solve it as a class. Have students explain their thinking and demonstrate how they would solve the problem. Discuss more than one method. Some may solve it in their heads, while others choose to use counters, the *200 Chart,* base-ten pieces, or paper and pencil. Some students may suggest using the calculator. Distribute calculators and introduce keystrokes for subtraction.

Ask:

- *Pretend you have $2.00 to spend at the snack shop. What would you order?*

List orders on the board. As a class determine the amount of change each student would receive. Encourage students to share their strategies.

After the class solves a few problems, summarize their work by discussing the steps students may have taken as they solved each one. The steps may include: locating or choosing items on the menu, estimating whether there is enough money, calculating the total, and finding the amount of change or the amount of money needed.

Have students work in pairs to solve the problems on the *Snack Shop Bills 2* Activity Pages. Allow time for student pairs to share their solutions with the class.

Name _____ Date _____

## Snack Shop Bills 2

| You have $1.75. Do you have enough money to buy the items listed below? | You have 40¢. Do you have enough money to buy the items listed below? |
|---|---|
| Estimate: yes ___ no ___ | Estimate: yes ___ no ___ |
| Actual: | Actual: |
| Peanut Butter and Crackers _____ | Bagel _____ |
| Chicken Noodle Soup _____ | Pretzels _____ |
| Small Milk _____ | |
| Total: _____ | Total: _____ |
| How much more money do you need? _____ or How much change will you get? _____ | How much more money do you need? _____ or How much change will you get? _____ |

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*Student Guide* - page 306 *(Answers on p. 82)*

Name _____ Date _____

| You have 80¢. Do you have enough money to buy the items listed below? | You have 94¢. Do you have enough money to buy the items listed below? |
|---|---|
| Estimate: yes ___ no ___ | Estimate: yes ___ no ___ |
| Actual: | Actual: |
| Chicken Noodle Soup _____ | Taco _____ |
| Carrot Sticks _____ | Brownie _____ |
| | Small Milk _____ |
| Total: _____ | Total: _____ |
| How much more money do you need? _____ or How much change will you get? _____ | How much more money do you need? _____ or How much change will you get? _____ |

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*Student Guide* - page 307 *(Answers on p. 82)*

Name _____ Date _____

Choose items from the menu to complete the bill below. Find the amount of change you should receive. Show how you solved the problem in the space provided.

| Shooting Star Snack Shop<br>You have up to $1.50 to spend. | | Customer's Name: |
|---|---|---|
| Item | Price(¢) | |
| _____ | _____ | _____ |
| _____ | _____ | |
| _____ | _____ | |
| _____ | _____ | |
| _____ | _____ | Change due |
| | Total _____ | _____ |

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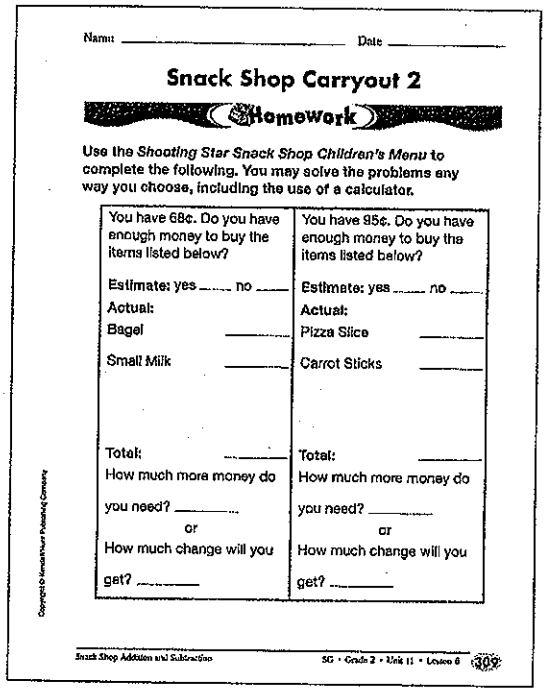
*Student Guide* - page 308 *(Answers on p. 82)*



Name _____ Date _____

## Snack Shop Carryout 2

Homework

Use the *Shooting Star Snack Shop Children's Menu* to complete the following. You may solve the problems any way you choose, including the use of a calculator.

| You have 68¢. Do you have enough money to buy the items listed below? | You have 95¢. Do you have enough money to buy the items listed below? |
|---|---|
| Estimate: yes ____ no ____ | Estimate: yes ____ no ____ |
| Actual: | Actual: |
| Bagel _____ | Pizza Slice _____ |
| Small Milk _____ | Carrot Sticks _____ |
| Total: _____ | Total: _____ |
| How much more money do you need? _____ or How much change will you get? _____ | How much more money do you need? _____ or How much change will you get? _____ |

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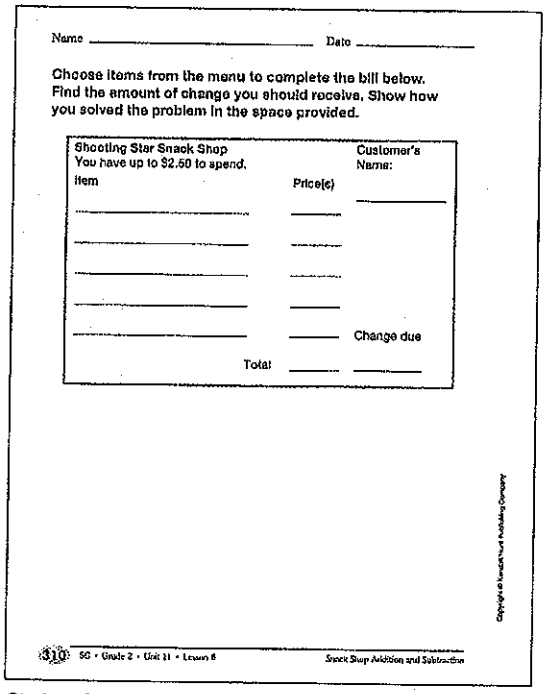
*Student Guide* - page 309 *(Answers on p. 83)*



Name _____ Date _____

Choose items from the menu to complete the bill below. Find the amount of change you should receive. Show how you solved the problem in the space provided.

| Shooting Star Snack Shop<br>You have up to $2.50 to spend. | | Customer's Name: |
|---|---|---|
| Item | Price(¢) | _____ |
| _____ | _____ | |
| _____ | _____ | |
| _____ | _____ | |
| _____ | _____ | |
| _____ | _____ | Change due |
| | Total _____ | _____ |

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*Student Guide* - page 310 *(Answers on p. 83)*

## Homework and Practice

- Assign the *Snack Shop Carryout 2* Homework Pages to give students more practice adding and subtracting priced items. Children need to take home the *Shooting Star Snack Shop Children's Menu* to complete this assignment.
- For DPP item R students write stories for number sentences.

## Assessment

- Observe students as they solve the problems in the *Student Guide.* Use the *Observational Assessment Record* to document students' abilities to solve problems involving money.
- Throughout this unit, students explored many methods of finding the answers to subtraction problems. Assess whether your students can solve a problem in more than one way. Have many types of math tools available for students such as counters, calculators, base-ten pieces, *200 Charts,* and paper and pencil. Write a subtraction problem on the board and ask students to solve it two different ways. They can use pictures and words to describe their solution paths.
- Use DPP item Q and the *Observational Assessment Record* to document students' fluency with the subtraction facts for Group A.
- Transfer appropriate documentation from the Unit 11 *Observational Assessment Record* to students' *Individual Assessment Record Sheets.*

# Exemplars® Standards-Based Math Rubric*

| | Problem Solving | Reasoning and Proof | Communication | Connections | Representation |
|---|---|---|---|---|---|
| **Novice** | No strategy is chosen, or a strategy is chosen that will not lead to a solution.<br><br>Little or no evidence of engagement in the task present. | Arguments are made with no mathematical basis.<br><br>No correct reasoning nor justification for reasoning is present. | No awareness of audience or purpose is communicated.<br>or<br>Little or no communication of an approach is evident<br>or<br>Everyday, familiar language is used to communicate ideas. | No connections are made. | No attempt is made to construct mathematical representations. |
| **Apprentice** | A partially correct strategy is chosen, or a correct strategy for only solving part of the task is chosen.<br><br>Evidence of drawing on some previous knowledge is present, showing some relevant engagement in the task. | Arguments are made with some mathematical basis.<br><br>Some correct reasoning or justification for reasoning is present with trial and error, or unsystematic trying of several cases. | Some awareness of audience or purpose is communicated, and may take place in the form of paraphrasing of the task.<br>or<br>Some communication of an approach is evident through verbal/written accounts and explanations, use of diagrams or objects, writing, and using mathematical symbols.<br>or<br>Some formal math language is used, and examples are provided to communicate ideas. | Some attempt to relate the task to other subjects or to own interests and experiences is made. | An attempt is made to construct mathematical representations to record and communicate problem solving. |

*Based on revised NCTM standards.

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# Exemplars® Standards-Based Math Rubric (cont.)*

| | Problem Solving | Reasoning and Proof | Communication | Connections | Representation |
|---|---|---|---|---|---|
| **Practitioner** | A correct strategy is chosen based on mathematical situation in the task.<br><br>Planning or monitoring of strategy is evident.<br><br>Evidence of solidifying prior knowledge and applying it to the problem solving situation is present.<br><br>Note: The practitioner must achieve a correct answer. | Arguments are constructed with adequate mathematical basis.<br><br>A systematic approach and/or justification of correct reasoning is present. This may lead to...<br>• clarification of the task.<br>• exploration of mathematical phenomenon.<br>• noting patterns, structures and regularities. | A sense of audience or purpose is communicated.<br>and/or<br>Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response.<br><br>Formal math language is used throughout the solution to share and clarify ideas. | Mathematical connections or observations are recognized. | Appropriate and accurate mathematical representations are constructed and refined to solve problems or portray solutions. |
| **Expert** | An efficient strategy is chosen and progress towards a solution is evaluated.<br><br>Adjustments in strategy, if necessary, are made along the way, and / or alternative strategies are considered.<br><br>Evidence of analyzing the situation in mathematical terms, and extending prior knowledge is present.<br><br>Note: The expert must achieve a correct answer. | Deductive arguments are used to justify decisions and may result in formal proofs.<br><br>Evidence is used to justify and support decisions made and conclusions reached. This may lead to...<br>• testing and accepting or rejecting of a hypothesis or conjecture.<br>• explanation of phenomenon.<br>• generalizing and extending the solution to other cases. | A sense of audience and purpose is communicated.<br>and/or<br>Communication at the Practitioner level is achieved, and communication of argument is supported by mathematical properties.<br><br>Precise math language and symbolic notation are used to consolidate math thinking and to communicate ideas. | Mathematical connections or observations are used to extend the solution. | Abstract or symbolic mathematical representations are constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon. |

*Based on revised NCTM standards.

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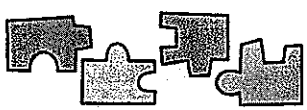
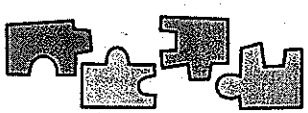
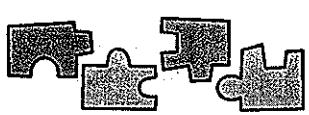
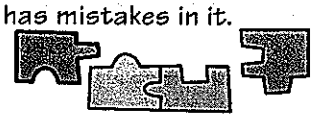
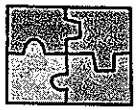
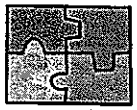
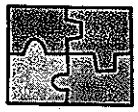
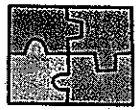
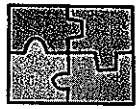
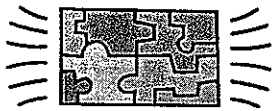
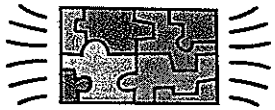
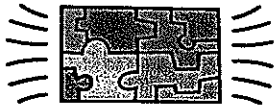
# Exemplars® Jigsaw Student Rubric

| Level | Problem Solving | Reasoning and Proof | Communication | Connections | Representation |
|---|---|---|---|---|---|
| **Novice**<br>Makes an effort<br>No or little understanding | I did not understand the problem. | My math thinking is not correct.  | I used no math language and/or math notation.  | I did not notice anything about the problem or the numbers in my work.  | I did not use a math representation to help solve the problem and explain my work. |
| **Apprentice**<br>Ok, good try<br>Unclear if student understands | I only understand part of the problem. My strategy works for part of the problem. | Some of my math thinking is correct. | I used some math language and/or math notation. | I tried to notice something, but it is not about the math in the problem. | I tried to use math representation to help solve the problem and explain my work, but it has mistakes in it.  |
| **Practitioner**<br>Excellent<br>Clear<br>Strong understanding<br>Meets the standard | I understand the problem and my strategy works. My answer is correct.  | All of my math thinking is correct.  | I used math language and/or math notation throughout my work.  | I noticed something about my math work.  | I made a math representation to help solve the problem and explain my work, and it is labeled and correct.  |
| **Expert**<br>Wow, awesome!<br>Exceptional understanding! | I understand the problem. I used a rule, and/or verified that my strategy is correct.  | I showed that I knew more about a math idea that I used in my plan. Or, I explained my rule.  | I used a lot of specific math language and/or notation throughout my work. | I noticed something in my work, and used that to extend my answer and/or I showed how this problem is like another problem. | I used another math representation to help solve the problem and explain my work in another way.  |



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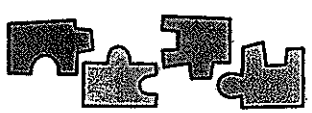
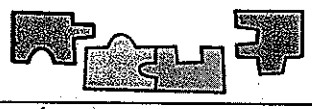
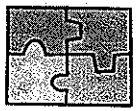
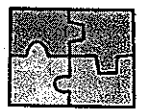
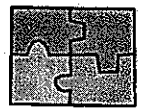
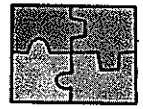
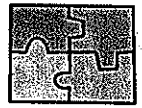
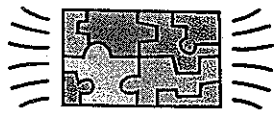
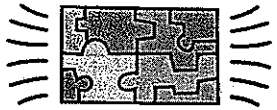
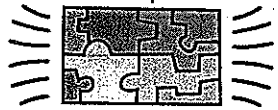
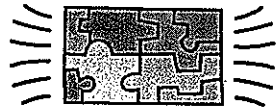
# Rúbrica Rompecabezas de Exemplars® para Estudiantes

| Nivel | Solución de Problemas | Razonamiento y Pruebas | Communicación | Conexiones | Representación |
|---|---|---|---|---|---|
| **Novato**<br>Hace un esfuerzo<br>Ninguna o poca comprensión | No comprendí el problema. | Mi razonamiento matemático no es correcto. | No usé ni lenguaje matemático ni anotación matemática. | No observé nada sobre el problema ni los números en mi trabajo.  | No usé una representación matemática para ayudar a solucionar el problema ni para explicar mi trabajo. |
| **Aprendiz**<br>Está bien, un buen esfuerzo<br>No está claro si el estudiante comprende el problema o no | Comprendo sólo una parte del problema. Mi estrategia funciona para parte del problema. | Parte de mi razonamiento matemático es correcto. | Usé algo del lenguaje matemático y/o anotación matemática.  | Intenté observar algo, pero no es sobre las matemáticas en el problema. | Intenté usar una representación matemática para ayudar a solucionar el problema y explicar mi trabajo, pero hay errores. |
| **Practicante**<br>Excelente<br>Claro<br>Comprensión fuerte<br>Llega al estandard | Comprendo el problema y mi estrategia funciona. Mi respuesta es correcta.  | Todo mi pensamiento matemático es correcto.  | Usé lenguaje matemático y/o anotación matemática en todo mi trabajo..  | Observé algo sobre mi trabajo matemático.  | Hice una representación matemática para ayudar a solucionar el problema y explicar mi trabajo y está claramente indicada y correcta.  |
| **Experto**<br>¡Wow! ¡Qué chévere!<br>¡Comprensión excepcional! | Comprendo el problema. Usé una regla y/o verifiqué que mi estrategia es correcta.  | Demostré que sabía más sobre una idea matemática que lo que usé en mi plan. O, expliqué mi regla. | Usé mucho lenguaje matemático específico y/o anotaciones en todo mi trabajo.  | Observé algo en mi trabajo y lo usé para extender mi respuesta y/o mostré cómo este problema se parece a otro problema.  | Usé otra representación matemática para ayudar a solucionar el problema y explicar mi trabajo de otra manera.  |



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