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INNOVATIONS

Learning Lab #4

Agenda

- *Begin with Breakfast*
- Investigation: Mental Math Strings
  - Video Analysis
- Operations & Number Stories
  - Problem Situations
  - Video Analysis
- High-Impact Strategies for Mathematics
  - Sharing multiple strategies or solutions without comment
- Big Ideas of Operations
- High-Quality Books to Spark Mathematical Thinking & Action
- *Break for Lunch*
- Evidence of Thinking in Problem Solutions
  - Student Work Samples
  - Lesson Analysis

Content Focus

- October: *Numerosity & Number Sense*
- November: *Counting, Cardinality & Number Sense*
- January: *Number Composition*

Strategy Focus

- October: *Turn & Talk*
- November & January: *Learners rephrase other learners' thinking*

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Focus for Today

- Content: ***Number Operations***
- Strategy: ***Sharing multiple solutions or strategies without comment***

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Basic Types of Addition & Subtraction Number Stories

- Change Situations
  - Join
  - Separate
- Relationships
  - Compare
  - Part-Part-Whole

By varying the unknown within each type of number story, many different problem situations can be constructed.

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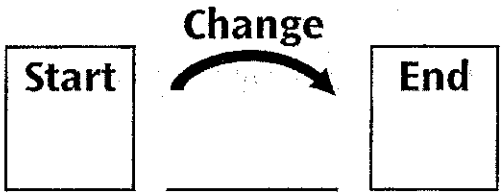
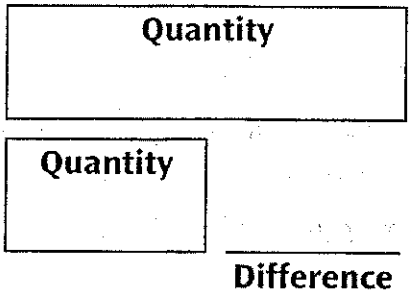
Change Situation



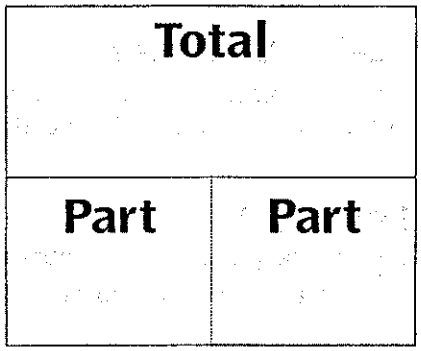
- Result Unknown - *How many in the end?*
- Change Unknown - *How many were added or taken away?*
- Start Unknown - *How many were there at first?*

Comparison Situation



- Difference Unknown - *How many more or less?*
- Quantity Unknown - *How many in the set?*

Part-Part-Whole Situation



- Whole Unknown - *How many altogether?*
- Part Unknown - *How many to complete the whole?*

Math at the Bakery

In your small group:

- Write a number story for your assigned problem type
  - Use the numbers 5, 8, 13
- Brainstorm at least two likely strategies <u>your students</u> would use to solve it

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Children's Solution Strategies

- Direct Modeling Strategies
  - Use of objects (manipulatives), fingers or drawings to directly model the action or relationship described in the problem
- Counting Strategies
  - Use the counting sequence itself to figure out the solution
  - Usually involves a strategy to keep track of counts
- Derived Facts Strategies
  - Use "friendly" numbers first: doubles, sums of tens
  - Over time, children learn many number facts at recall level

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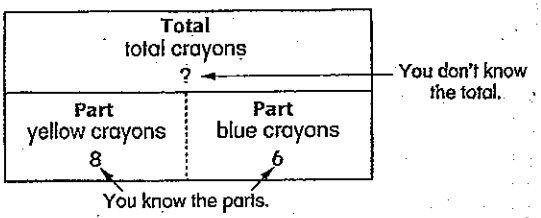
Number Stories and Situation Diagrams

Read It Together

Number stories are stories that use numbers.
One way to solve number stories is to use diagrams.

Some number stories are about a total and its parts.
You can use a **parts-and-total diagram** to help you solve them.

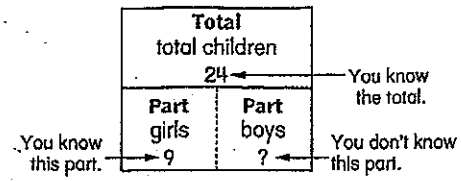
There are 8 yellow crayons and 6 blue crayons.
How many crayons are there in all?



Add the parts to find the total.
Number model: 8 + 6 = **14**
There are **14** crayons in all.

Sometimes you need to find one of the parts.

There are 24 children on a bus.
9 children are girls. How many children are boys?



You can subtract to solve the problem.
Subtract the part you know from the total.
The answer is the other part.
Number model: 24 − 9 = **15** **15** boys on the bus

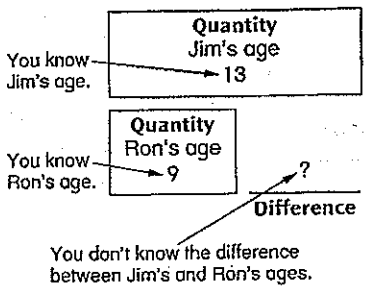
You can also add up to solve the problem.
Start with the part you know. Add up to the total.
The amount you add up is the other part.

+1 +10 +4
9 → 10 → 20 → 24

1 + 10 + 4 = **15**
Number model: 9 + **15** = 24 **15** boys on the bus

Some number stories are about comparisons. You can use a **comparison diagram** to help you solve them.

Jim is 13 years old. Ron is 9 years old.
How many years older is Jim than Ron?



You can subtract to find the difference.
Start with the larger number.
Subtract the smaller number.
The answer is the difference.

Number model: 13 − 9 = **4**

Jim is **4** years older than Ron.

You can also add up to find the difference.
Start with the smaller number.
Add up to the larger number.
The amount you add up is the difference.

+1 +3
9 → 10 → 13

1 + 3 = **4**

Number model: 9 + **4** = 13

Jim is **4** years older than Ron.

Tell your partner a comparison number story.

You can use a diagram to help you solve a number story about groups with equal numbers of objects.

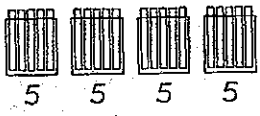
- Mia has 4 packs of gum.
  There are 5 sticks of gum in each pack.
  How many sticks of gum are there in all?

Number of groups	Number of objects in each group	Total number of objects
packs	*sticks per pack*	*sticks in all*
4	5	?
You know the number of groups.	You know the number of objects in each group.	You don't know the total number of objects.

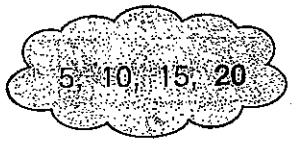
Number model: $4 \times 5 = ?$

There are many ways to solve the problem.

- You can draw a picture.



- You can skip count by 5s.



- You can add 5 four times.
  $5 + 5 + 5 + 5 = 20$

Number model: $4 \times 5 = 20$

There are 20 sticks of gum in all.

Equal shares number stories are about dividing groups of objects into parts called **shares.** You can use a diagram to help you solve an equal share story.

- There are 24 marbles and 3 children.
  Each child should get the same number of marbles.
  How many marbles should each child get?

Number of shares	Number of objects in each share	Total number of objects
children	*marbles per child*	*marbles in all*
3	?	24
You know the number of shares.	You don't know the number of objects in each share.	You know the total number of objects.

Number models: $3 \times ? = 24$
$24 \div 3 = ?$

- One way to solve the problem is to draw a picture.

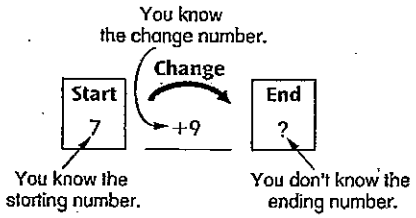
- Another way is to use counters. Deal out 24 counters among 3 equal shares to see how many counters are in each share.

Number models: $3 \times 8 = 24$
$24 \div 3 = 8$

Each child has 8 marbles.

In change stories, the number you start with changes to more or changes to less. You can use a **change diagram** to help you solve a change-to-more story.

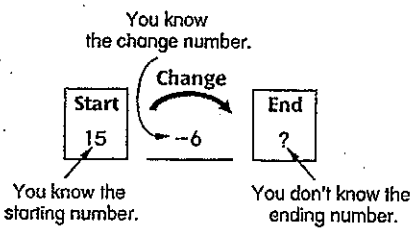
Britney had 7 shells.
She found 9 more shells.
How many shells does Britney have in all?



Number model: 7 + 9 = 16
Britney has 16 shells in all.

You can use a change diagram to help you solve a change-to-less story.

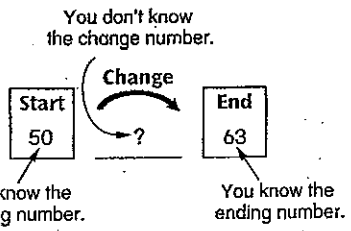
There are 15 children on a bus.
6 children get off the bus.
How many children are left on the bus?



Number model: 15 − 6 = 9
There are 9 children left on the bus.

Sometimes you need to find the change in a change story. You can use a change diagram to help you solve this kind of number story.

The morning temperature was 50°F.
The afternoon temperature was 63°F.
What was the temperature change?



Number model: 50 + 13 = 63
The temperature change was +13°F.

Try It Together

Take turns with a partner making up and solving number stories.

Everyday Mathematics

The University of Chicago School Mathematics Project

My Reference Book

McGraw-Hill Wright Group

Direct Modeling Strategies

Problem	*Strategy Description*
Join (Result Unknown) Ellen had 3 tomatoes. She picked 5 more tomatoes. How many tomatoes does Ellen have now?	*Joining All* A set of 3 objects and a set of 5 objects are constructed. The sets are joined and the union of the two sets is counted.
Join (Change Unknown) Chuck had 3 peanuts. Clara gave him some more peanuts. Now Chuck has 8 peanuts. How many peanuts did Clara give him?	*Joining To* A set of 3 objects is constructed. Objects are added to this set until there is a total of 8 objects. The answer is found by counting the number of objects added.
Separate (Result Unknown) There were 8 seals playing. 3 seals swam away. How many seals were still playing?	*Separating From* A set of 8 objects is constructed. 3 objects are removed. The answer is the number of remaining objects.
Separate (Change Unknown) There were 8 people on the bus. Some people got off. Now there are 3 people on the bus. How many people got off the bus?	*Separating To* A set of 8 objects is counted out. Objects are removed from it until the number of objects remaining is equal to 3. The answer is the number of objects removed.
Compare (Difference Unknown) Megan has 3 stickers. Randy has 8 stickers. How many more stickers does Randy have than Megan?	*Matching* A set of 3 objects and a set of 8 objects are matched 1-to-1 until one set is used up. The answer is the number of unmatched objects remaining in the larger set.
Join (Start Unknown) Deborah had some books. She went to the library and got 3 more books. Now she has 8 books altogether. How many books did she have to start with?	*Trial and Error* A set of objects is constructed. A set of 3 objects is added to the set, and the resulting set is counted. If the final count is 8, then the number of objects in the initial set is the answer. If it is not 8, a different initial set is tried.

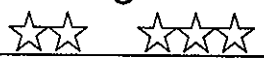
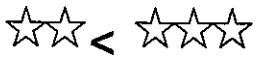
Reprinted from *Children's Mathematics: Cognitively Guided Instruction* by Carpenter, Fennema, Franke, Levi, & Empson. Heinemann (1999).

Counting Strategies

Problem	*Strategy Description*
Join (Result Unknown) Ellen had 3 tomatoes. She picked 5 more tomatoes. How many tomatoes does she have now?	*Counting On From First* The counting sequence begins with 3 and continues on 5 more counts. The answer is the last number in the counting sequence.
Join (Result Unknown) Ellen had 3 tomatoes. She picked 5 more tomatoes. How many tomatoes does she have now?	*Counting On From Larger* The counting sequence begins with 5 and continues on 3 more counts. The answer is the last number in the counting sequence.
Join (Change Unknown) Chuck had 3 peanuts. Clara gave him some more peanuts. Now Chuck has 8 peanuts. How many peanuts did Clara give to him?	*Counting On To* A forward counting sequence starts from 3 and continues until 8 is reached. The answer is the number of counting words in the sequence.
Separate (Result Unknown) There were 8 seals playing. 3 seals swam away. How many seals were still playing?	*Counting Down* A backward counting sequence is initiated from 8. The sequence continues for 3 more counts. The last number in the counting sequence is the answer.
Separate (Change Unknown) There were 8 people on the bus. Some people got off. Now there are 3 people on the bus. How many people got off the bus?	*Counting Down To* A backward counting sequence starts from 8 and continues until 3 is reached. The answer is the number of words in the counting sequence.

Reprinted from *Children's Mathematics: Cognitively Guided Instruction* by Carpenter, Fennema, Franke, Levi, & Empson. Heinemann (1999).

Big Ideas about Operations

Topic	Big Ideas	Examples
Joining & Separating 	•A collection can be made larger by adding items to it and made smaller by taking some away from it.	•*You have 2 balls and I have 3 balls. How many balls do we have altogether?* •*You had 60 cards, and you gave your friend 5. How many do you have now?*
Grouping & Partitioning 	•One can quantify a collection by grouping items into equal sets.	•*Chris has 2 boxes of crayons with 4 in each box. How many crayons does Chris have altogether?* •*There are 20 children in the 2nd grade class. Sandy brings 40 cookies so each child can have two.* •*How many hands does it take to show 20 fingers?* •*How can 3 children share 9 toy cars fairly?*
Composing & Decomposing 	•A quantity (whole) can be "broken apart" (decomposed) into parts, and the parts can be combined (composed) to form the whole.	•*How many ways can you show 5 with fingers on both hands?* • *100 can be 50 & 50 or 70 & 30 or 90 & 10.*
Comparing 	•When comparing quantities, there are two possible results – equality or greater than/less than.	•*I have a handful of raisins; Chris has a bowl-ful. Chris has more!* •*I have 1 pear and 1 peach; you have 2 apples. We have the same number of fruits.* •*Avery has 3 dirty plates, and Tracy has 4 dirty bowls. Who has fewer dishes to wash?* •*There are 6 fish and 3 snails in our aquarium. We have twice as many fish as snails.*
Solving Problems ?	•The four arithmetic operations (addition, subtraction, multiplication & division) are tools for solving problems about numbers. •In order to choose which operation to use, the solver must understand what is happening in the problem situation.	•*There is usually more than one way to solve the same problem. For example, subtraction or counting up are equally valid ways to find the difference between two numbers.* •*All word problems tell a story.*

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High Quality Books to Spark Children's Thinking About Number Composition & Operations

Anno, M. *Anno's Counting Book.* HarperCollins Publishing, 1975.

Baker, K. *Quack and Count.* Voyager Books, 1999.

Bang, M. *Ten, Nine, Eight.* Greenwillow Books, 1983.
Tambien en espanol: Bang, M. *Diez, Nueve, Ocho.* Mulberry.

Crews, D. *Ten Black Dots.* Greenwillow Books, 1986.0
Tambien en espanol: Crews, D. *Los Diez Puntos Negros.* Greenwillow Books.

Demi. *One grain of rice: A mathematical folktale.* Scholastic, 1997.

Ehlert, L. *Fish Eyes.* Sandpiper, 1990.

Fleming, D. *Count!* Henry Holt, 1992.

Giganti, P. *Each orange had 8 slices: A counting book.* Greenwillow, 1992.
Giganti, P. *How many snails? A counting book.* Greenwillow, 1988

Harris, T. *Splitting the Herd.* Millbrook, 2008.

Harshman, M. *Only one.* Scholastic, 1993.

Hoban, T. *26 letter and 99 cents.* Greenwillow Books, 1987.
Hoban, T. *Count and See.* Simon & Schuster, 1972
Hoban, T. *Let's Count.* Greenwillow Books, 1999.
Hoban, T. *More, Fewer, Less.* Greenwillow Books, 1998.

Hong, L.T. *Two of everything.* Albert Whitman & Company, 1993.

Hopkins, L.B. & Barbour, K. *Marvelous Math: A Book of Poems.* Aladdin, 1997.

Hutchins, P. *The Doorbell Rang.* Mulberry, 1986.
Tambien en Espanol: Hutchins, P. *Llaman a la Puerta.* Mulberry.

Jonas, A. *Splash!* Greenwillow Books, 1995.

Kellogg, S. *How Much is a Million?* Harper Collins Publishing, 1985.

Lewis, J.P. *Arithmetickle.* Harcourt, 2002.

Mahy, M. *17 Kings and 42 Elephants.* Dial Books, 1987.

McMillan, B. *Eating fractions.* Scholastic, 1991.

Merriam, E.*12 Ways to Get to 11.* Aladdin Paperbacks, 1993.

Neuschwander, C. *Amanda Bean's Amazing Dream.* Scholastic, 1998.

Nolan, H. *How much, how many, how far, how heavy, how long, how tall is 1000?* Scholastic, 1995.

Princzes, E. *One Hundred Hungry Ants.* Houghton Mifflin Company,1993.
Princzes, E. *A Remeinder of One.* Houghton Mifflin Company,1995.

Sayre, A.P. & Sayre, J. *One is a Snail, Ten is a Crab.* Candlewick, 2003.

Schlein, M. *More than one.* Scholastic,1996.

Tang, G. *The best of times.* Scholastic, 2002. (ages 7-12)

Tang, G. *The Grapes of Math.* Scholastic, 2001. (ages 7-12)
Tambien en Espanol: Tang,G. *Come Una y Cuenta 20.* Everest

Tang, G. *Math appeal: mind-stretching math riddles..* Scholastic, 2003. (ages 7-12)

Tang, G. *Math fables: lessons that count.* Scholastic, 2004. (ages 3- 6)

Tang, G. *Math for All Seasons.* Scholastic, 2002. (ages 5-8)
Tambien en Espanol: Tang, G. *Un, Dos, Tres, El Ano se Fue.* Everest.

Tang, G. *Math potatoes: mind-stretching brain food.* Scholastic, 2005. (ages 8-13)

Tang, G. *Math-terpieces: the art of problem-solving.* Scholastic, 2003. (ages 5-12)

Walsh, E.S. *Mouse Count.* Harcourt, 1991
Tambien en Espanol: Walsh, E.S. *Cuenta Ratones.* Fondo de Cultura Economica.