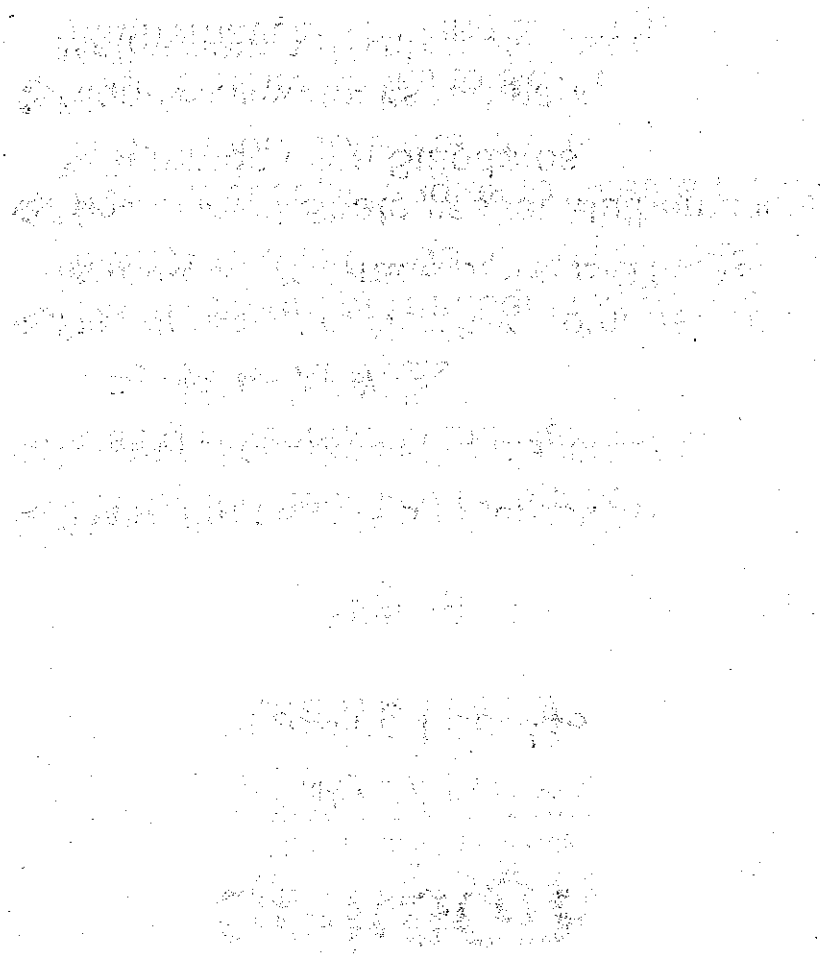
erikson
early mathematics education
INNOVATIONS

Learning Lab #3

Agenda

- "What the heck is a rekenrek?"
- Using rekenreks in the classroom
  - Video Analysis
- Development of Number Composition from Age 3 to Grade 3: the BIG IDEAS
- High-Impact Strategies for Mathematics & Common Core Standards
- High-Quality Books to Spark Mathematical Thinking & Action



by Barbara Blanke (2008)

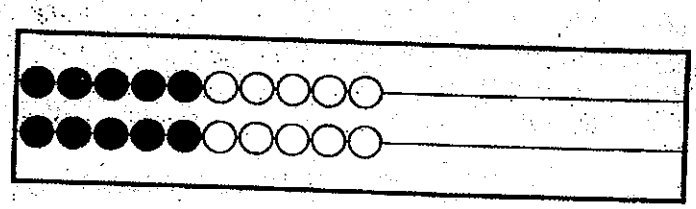
Using the Rekenrek as a Visual Model for Strategic Reasoning in Mathematics

"...materials cannot transmit knowledge: the learner must construct the relationships" Gravemeijer, 1991

What is a Rekenrek?

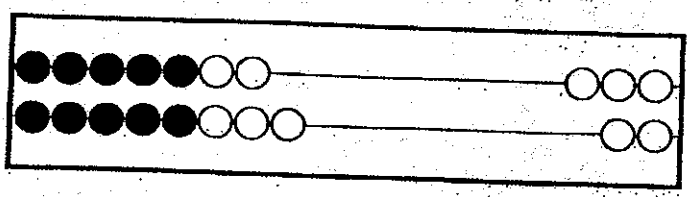
Directly translated, rekenrek means calculating frame, or arithmetic rack. Adrian Treffers, a mathematics curriculum researcher at the Freudenthal Institute in Holland, designed it to support the natural mathematical development of children and to help them generate a variety of addition and subtraction strategies, including doubles plus or minus 1, making 10's, and compensation. Students can use the rekenrek to develop computation skills or solve contextual problems. Once children understand the operations of addition and subtraction, and can model various situations, it is important that they automatize the basic facts by finding and using patterns and relationships. Unlike drill and practice worksheets and flashcards, the rekenrek supports even the youngest learners with the visual models they need to discover number relationships and develop automaticity.

The rekenrek looks like an abacus, but it is not based on place value columns or used like an abacus. Instead, it features two rows of 10 beads, each broken into two sets of five, much like the ten frames used in Bridges in Mathematics.



The rekenrek

Like ten frames, this tool helps students see the quantity of five as a whole and develop strategies to solve equations like 5 + 2, 5 + 3, etc. The figure below shows how 7 + 8 would be set up on the rekenrek. In working with this model, children might find the total by adding 7 + 7 + 1, or 5 + 5 + 2 + 3, or 10 + 5. They could also choose to count on; but the rekenrek is likely to stretch children to see groups of five.



7 + 8

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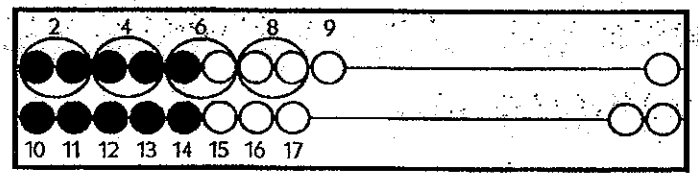
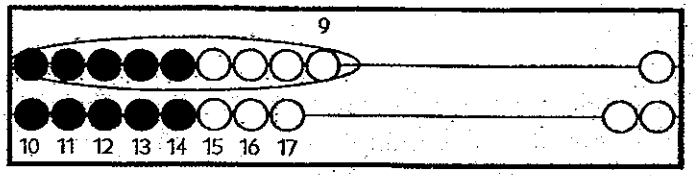
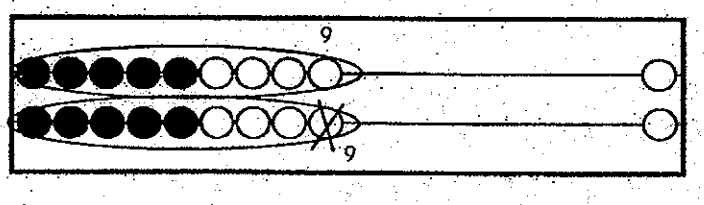
Here are some strategies we've seen students use for solving 9 + 8 on the rekenrek.



Hannah's Strategy: counts by 2's to 8 then counts on by 1's

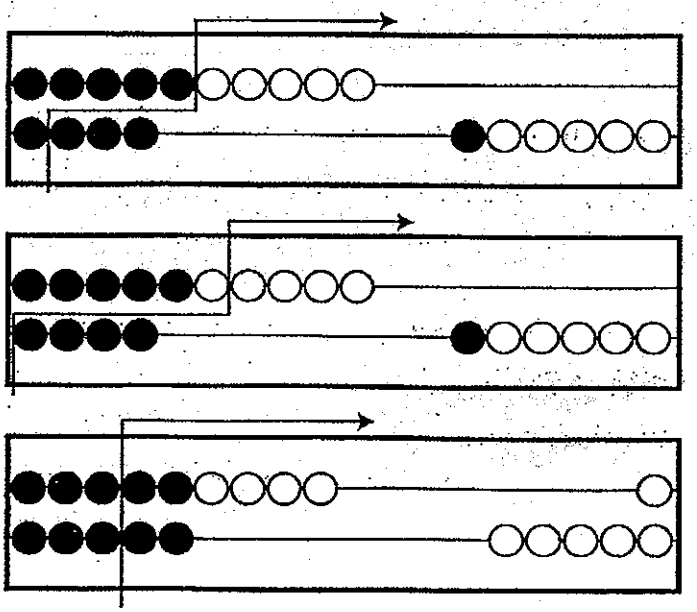


Ryan's Strategy: sees 9 and counts on by 1's



Ellie's Strategy: doubles minus 1. Shows 9 + 9 and removes 1.

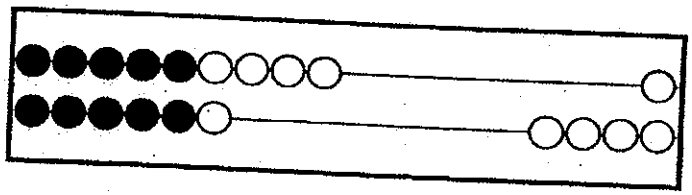
The rekenrek also allows students to develop a variety of subtraction strategies. Stanford professor Robert McKim (1980) states that, "the thinker who fluently produces a number of possible solutions to a problem is usually more successful than the thinker who settles for one solution". The figure below shows three different ways that children might solve 14 – 8 using the rekenrek.



Various solutions for 14 – 8

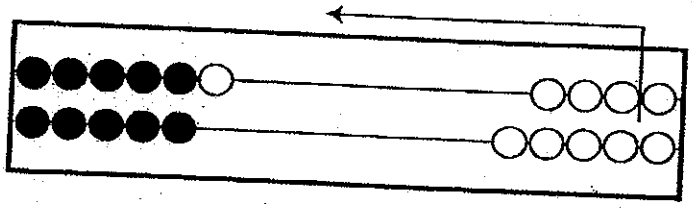
While students can use the rekrenrek to generate different strategies for solving basic facts, they can also use it to solve story problems such as the ones below.

> ***There** were 15 kids in the house. 9 were playing video games upstairs. How many were in the kitchen having snacks?*



A student might show the 9 children upstairs with 9 beads in the top row, and then count on additional beads in the bottom row until reaching 15. One can then see there were 6 children in the kitchen.

> ***There** were 6 kids doing math Home Connections upstairs. 5 kids were downstairs reading books. 3 more kids came home to study. How many were in the house?*



Various ways students solved this problem:

5 + 5 + 1 + 3 =

10 + 4 =

9 + 5 =

5 + 5 + 4 =

How to Get Started Using the Rekenrek

Using 5 and 10 as anchors for counting, adding, and subtracting is far more efficient than one-by-one counting. With the help of the rekenrek, kindergartners can start making the transition about the middle of the school year. The activities on the following pages show how. Each activity assumes familiarity with rekenrek operations in a previous activity.

Activity 1

What the Heck is a Rekenrek? Getting Familiar with the Calculating Frame

Grade Levels

K–3 & pre-K!

Skills

- ★ Familiarization with the calculating frame
- ★ Subitize 5 and 10
- ★ Develop vocabulary for rekenrek use

You Will Need

- ★ One rekenrek for teacher demonstration
- ★ Something to "hide" the rekenrek's bottom row of beads (a folded 8" × 5" index card or piece of construction paper, or a small piece of fabric)

Instructions for What the Heck is a Rekenrek

As students watch, cover the bottom row of beads on your rekenrek with a folded index card or construction paper or fabric. Then slide all the beads in the visible row over to the far right. (All beads over to the right is the "start" position for this activity and others in this collection.)

Ask your students what they notice about today's mathematical tool.

Teacher *What do you notice about the rekenrek?*

Jose *I see red and white beads.*

Ellie *I think there are ten.*

Teacher *How can we prove or disprove Ellie's thought?*

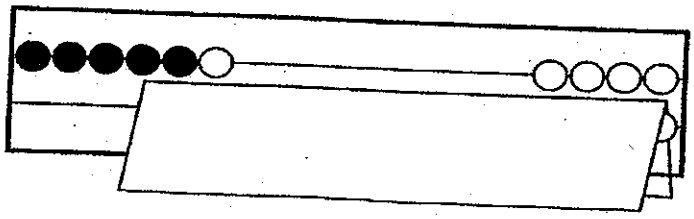
Hannah *I counted five red and five white.*

Daniel *That is right, because I saw that half were red and half were white.*

Marshall *5 plus 5 makes 10 total!.*

Explain that you're going to slide some beads on your rekenrek from one side of the wire to the other. Ask students to watch carefully and be ready to share how many you move.

- Slide 3 red beads to the left. Ask, "How many red beads do you see?"
- Return the beads to the right side of the rekenrek.
- Next slide 5 red beads to the left. Say, "Think provately. How many beads did I move? Quitely share with a friend why you think your number is right."
- Return the beads to the right side of the rekenrek.
- Slide over 6 beads. Ask "What do you notice?"

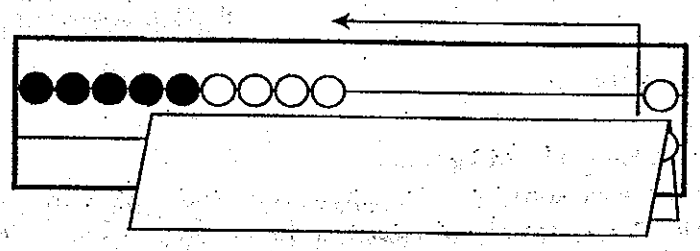


Activity 1 What the Heck is a Rekenrek? (cont.)

At this point let several children share how they can prove there are 6 beads... 3 and 3, counting by ones, five and 1 more, 1 and five more, 2 and 2 and 2 etc... Celebrate the multiple ways of "seeing" 6, especially the ones that did not involve one-by-one counting.

- Now slide 3 more beads over to the left for a total of 9. Ask, "How many do you see now?"

Keep soliciting different ways to prove there are 9 beads. Hopefully, someone will use the anchor of 10 saying something like, "I know there are 10 beads, so one less than 10 is 9" or, "I know there are 5, and 5 more would be 10, minus one is 9." Your goal is to look for non-counting strategies that assist students in subitizing 5 and 10. We want these numbers to become anchors for the students that they quickly visualize.



Activity 2

See and Slide Getting Efficient Using the Rekenrek

Grade Level
K-2

For Pre-K, use one row rekenrek and numbers 1-10 only.

Skills

- ★ Develop number sense strategies using 5 and 10 as anchor numbers
- ★ Visualize the numbers 1–20
- ★ Build numbers efficiently

You will need

- ★ One rekenrek for each child in the group
- ★ 20 Colored Popsicle sticks (numbers 1–10 in one color, numbers 11–20 in another color)
- ★ 1 container to hold popsicle sticks

Note This activity can be extended on a ten row rekenrek by using 100 colored popscicle sticks numbered from 1–100. Have students represent the numbers using the fewest possible bead moves.

Instructions for See and Slide

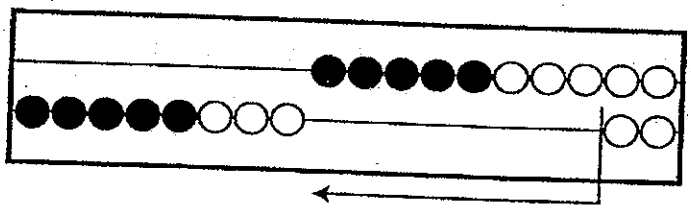
Make sure each child has a rekenrek with all beads moved to the right side. (Do not cover up any beads for this activity.)

Tell the students you are going to choose a number (from a container of popsicle sticks numbered 1–10). They are to "slide" that number of beads on the rekenrek, using only one move.

Teacher *I chose number 8. Think how you will move eight beads on the rekenrek in only one move. Now slide the beads.*

Mike *I slid 5 reds and 3 whites on the top row all in one move!*

Sandy *I did the same thing but on the bottom row.*



Pat *I moved 5 reds on top and 3 reds on the bottom.*

Merrie *Is that one move? I thought we had to use only one move.*

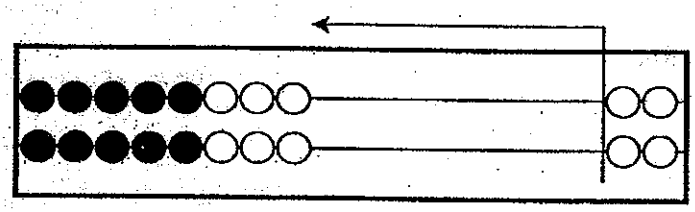
Pat *I used two fingers to make one move!*

Teacher *All of you "saw" the number eight and made one slide to show that number on your rekenrek.*

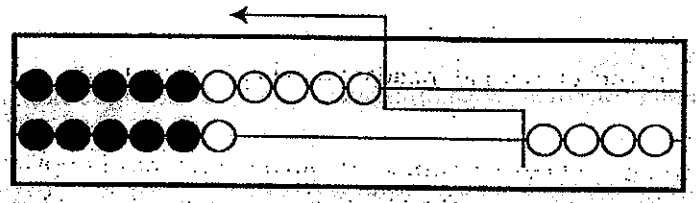
Activity 2 See and Slide! (cont.)

Now include the numbered sticks 11-20 in the container. Tell students they are to use no more than two slides to show numbers larger than 10.

> ***Teacher*** *I chose the number 16. Think about that number. How will it look when we move the beads? Using one or two moves, how can I show 16 on the rekenrek?*
>
> ***Henry*** *If you move 8 beads on each row, that will make 16.*
>
> ***Marcia*** *Move 10 on the top, 5 on the bottom, and 1 more on the bottom.*
>
> ***Mark*** *But that's three moves. You would need to move 10 on the top and 6 on the bottom.*



Doubles



Ten and More

Again, ask students to share their thinking. Continue the activity by asking students to choose and represent additional numbers. Discourage counting one by one by reminding them to use only one slide for numbers 1-10, and at most two slides for numbers 11-20.

Extra Activity

Finding Different Ways to Make a Given Number (Pre-K, K & 1st grade)

- Use 10-bead rekenrek. (Or, if you have a 20-bead rekenrek, cover the bottom row.)
- Begin by sliding the red beads to the left and the white beads to the right.
- Choose a number to build.
    - "Let's see how many ways we can build 4 by sliding beads from each side to the middle. "
    - "What if I slide 2 red beads from the left and 2 white beads from the right. Does that make 4 beads?"
    - "Can you think of another way to make 4?"
    - Record the different ways 4 can be built.
- This activity should be repeated many times using different numbers from 1-10.

Activity 4

It Takes Two to Build a Number Finding Missing Addends

Grade Level
K–2

Skills
- ★ Discover strategies to solve unknown or missing addend problems
- ★ Develop problem-solving strategies
- ★ Build fluency with facts to ten

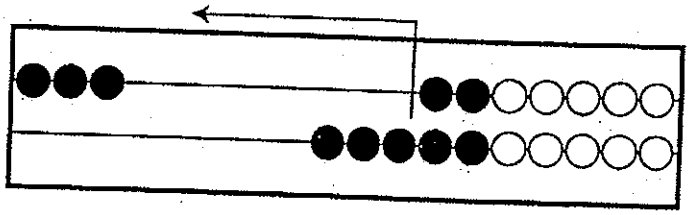
You Will Need
- ★ One rekenrek for every two students
- ★ A rekenrek for teacher demonstration
- ★ Student set of number cards 1–10 (page 25)
- ★ Student set of number cards 1–20 (pages 25–26, optional)

Note Rekenrek activities like this can be extended on the ten row rekenrek by using the 1–100 number cards, pages 25-31. Have students represent the numbers using the fewest possible bead moves.

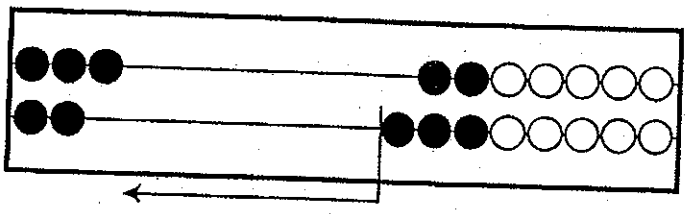
Instructions for It Takes Two to Build a Number

Ask a student to be your partner. Tell the class that you and your partner are going to build the number 5 as a team. You will be in charge of moving beads on the top row of the rekenrek and your student partner will move beads on the bottom row.

Teacher to Partner *I am going to slide 3 beads to the left on the top row. Now in one move, you slide beads on the bottom row to build the number 5.*



Shelby *I slid 2 beads on the bottom row with one move!*



Teacher *Now I would like all of you to pair up with a partner. I'll choose a number from the number cards. You will then build that number with your partner. (Begin with cards 1–10, or 1–20 if you are practicing facts to 20).*

Activity 4 It Takes Two to Build a Number (cont.)

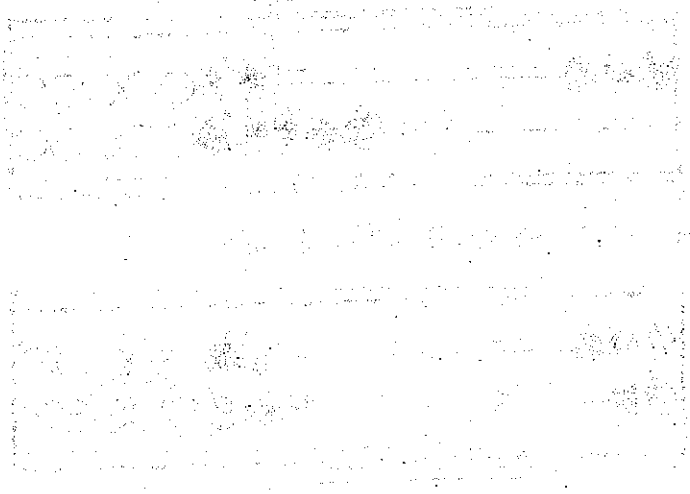
Choose a couple cards to further model this activity before inviting the class to build the numbers on their rekenreks. Tell students the number of beads on the top row should be less than the number chosen. For instance, if the number card chosen is 8, the first student might move 5 beads on the top. Her partner would then move 3 beads on the bottom.

Optional: Building numbers 11-20

Use the 11–20 number cards to practice facts to 20. Help students see that certain row combinations require a minimal number of beads on one row to build numbers larger than 10.

> ***Teacher*** *I'm going to move 4 beads on the top row. You slide beads on the second row to make the number 16.*
>
> ***Michael*** *That won't work. I don't have enough beads.*
>
> ***Sharon*** *You have to move more beads on the top.*
>
> ***Kelly*** *Move 6 beads on the top. Then the 10 on the bottom will make 16.*

Extension If you have a ten row rekenrek, challenge those students who are ready to represent numbers larger than 20. Have students work in pairs or small groups, taking turns drawing a number from the 1–100 number cards and building the number together or individually.



Activity 8

Target Think Addition

Grade Level
2–3

Skills

★ Use the "Think Addition" strategy to solve subtraction facts to 20

★ Combine numbers to find a target number

★ Decompose and compose numbers

You Will Need

★ Set of Number Cards 11-20 to use as Target Numbers (pages 25 and 26, run on cardstock and cut apart.)

★ Set of Number Cards 1-10 to use as Starting Numbers (pages 25 and 26, run on cardstock and cut apart.)

★ Two containers—one for holding "Starting" number cards 1-10, one for holding "target" number cards 11-20

★ One rekenrek per student

★ Optional rekenrek for teacher demonstration

Note Subtraction as "Think Addition" is a significant strategy for recalling subtraction facts.

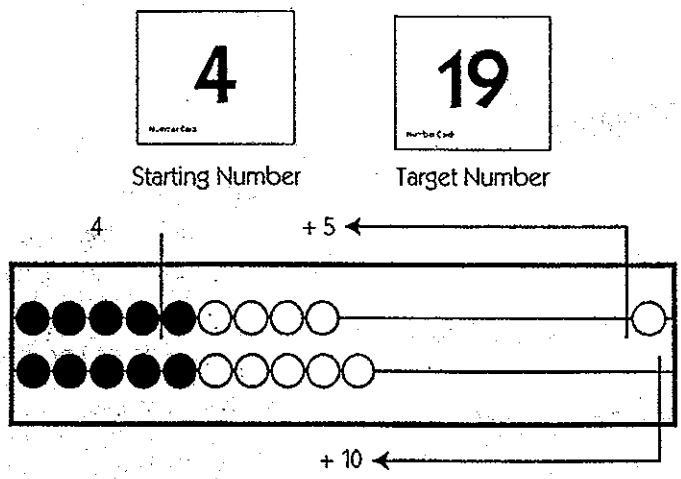
Instructions for Target Think Addition

- Draw a Starting Number Card from a container of number cards 1-10. Ask students to build the number on the rekenrek.
- Next draw a Target Number Card from a container of number cards 11-20. Tell students that number is the target they want to get to.
- Ask students to think of how many beads they must add to the Starting Number to equal the Target Number. Before they move any beads, ask students to share how many they will move. Then have them make the move to create the Target Number. Invite students to share their methods.

This mental activity leads to the use of a "think addition" strategy rather than a "count what is left" strategy when solving subtraction problems. It reinforces the concept of anchor numbers and seeing combinations rather than counting by ones.

> ***Teacher*** *I'm going to draw our Starting Number—it's 4. Build that number on your rekenrek.*
>
> ***Teacher*** *Now let's choose our Target Number. We drew 19! Think for a moment before you move any beads. How many will you need to move over so that you have the target number 19? Remember, we want to use as few moves as we can.*
>
> ***Charles*** *I would move the whole bottom 10 over first and that would make 14. Then I would move 5 more over on the top and that would make the target number 19.*
>
> ***Teacher*** *That's pretty good, only two moves. So how many beads did Charles move all together?*
>
> ***Terri*** *He moved 15. So we had to add 15 to 4 to hit the target 19. 4 + 15 is 19.*

Activity 8 Target Think Addition! (cont.)

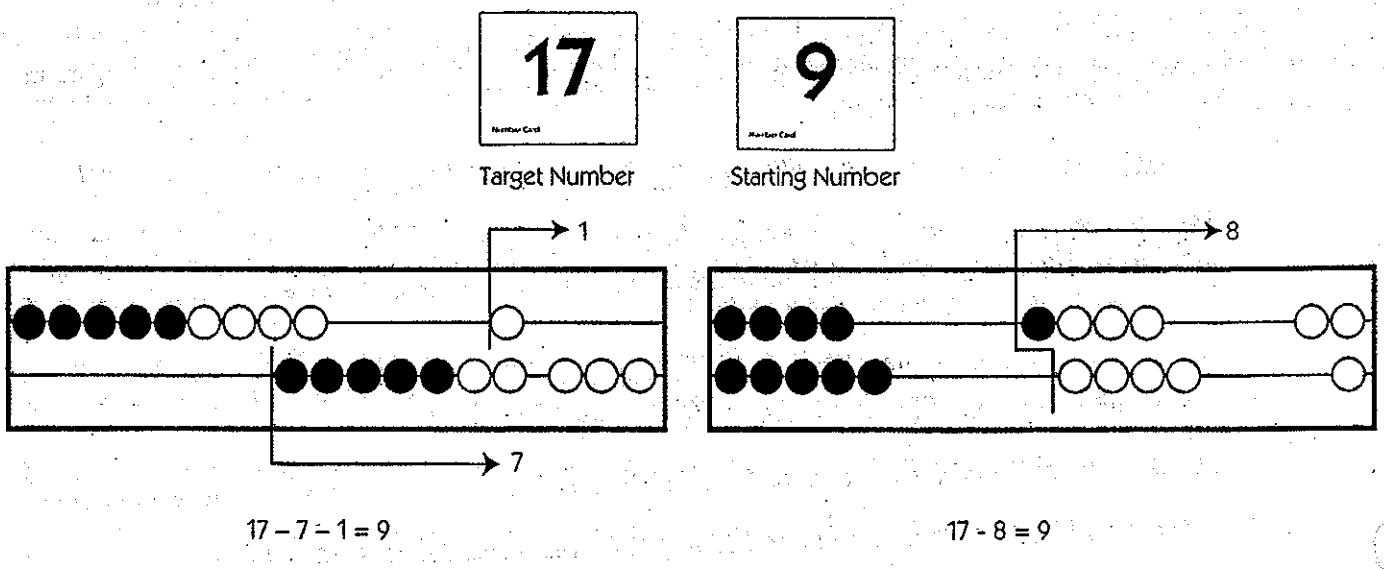


Repeat this activity several times with new Starting and Target numbers.

Extend to Subtraction

To move students into subtraction, draw the cards in a different order. Start by drawing a Target Number (11-20). Ask the class to build it on their rekenrek. Then draw a Starting Number Card (0-10). Ask students to think of how many beads they will need to remove from the Target Number (move back to the right) to get back to the Starting Number.

> ***Teacher*** *This time let's start with a Target Number. I've drawn a target of 17. Build that on the rekenrek. (Pause) Now let's draw a Starting Number—9. Think for a moment. How many beads will you need to remove from the Target Number to get back to the Starting Number 9?*
>
> ***Samantha*** *To get to 9 I'm going to have to move 7 back on the bottom and 1 more on top, that's 8 I have to move to get to 9. 17 - 8 is 9.*
>
> ***Kelly*** *I started with 17 as doubles plus 1. To get to 9, I took double 4s away. 17 - 8 is 9.*



Reprinted from *Young Mathematicians at Work: Constructing Number Sense, Addition, and Subtraction.* Catherin Twomey Fosnot & Maarten Dolk, Heinemann, 2001.

Using the Rekenrek *with Contexts and Routines*

Thus far, the examples of the *rekenrek* in action have all been devoid of context. We do not want to imply that context is not important or that the arithmetic rack cannot be used with context. On the contrary, it lends itself well to rich contexts and models.

In New York City, as in London and other parts of the world, double-decker buses are frequently used to take tourists on sight-seeing trips. Riders can choose whether to sit on the top, which is open to the air, or on the bottom, which is enclosed. The arithmetic rack is a great manipulative for children to use as a tool to solve situations like these:

> *Seventeen people get on an empty bus. The bus driver sees that nine sat downstairs. How many are up top?*
> *There are seven people up top and six below when the bus stops. Four more people get on. How many are on the bus now?*
> *Let's make our own bus storybooks.*

Bunk beds are another wonderful context easily modeled with the arithmetic rack. Anne Giordano and Carla Middough came up with this context and the children loved it. They told a group of first graders about some kids that had a pajama party. They were all on the bunk beds listening to music. Here again, scenarios of on and off, moving from top to bottom, were designed, providing rich contexts for addition and subtraction. Other situations in the lives of children can also be found and used to provide rich contexts for mathematizing.

The ideas behind the development of the arithmetic rack—supporting and stretching the invented strategies of children by using moveable objects, two sets of five (making ten) on each line—can also be employed with many classroom routines.

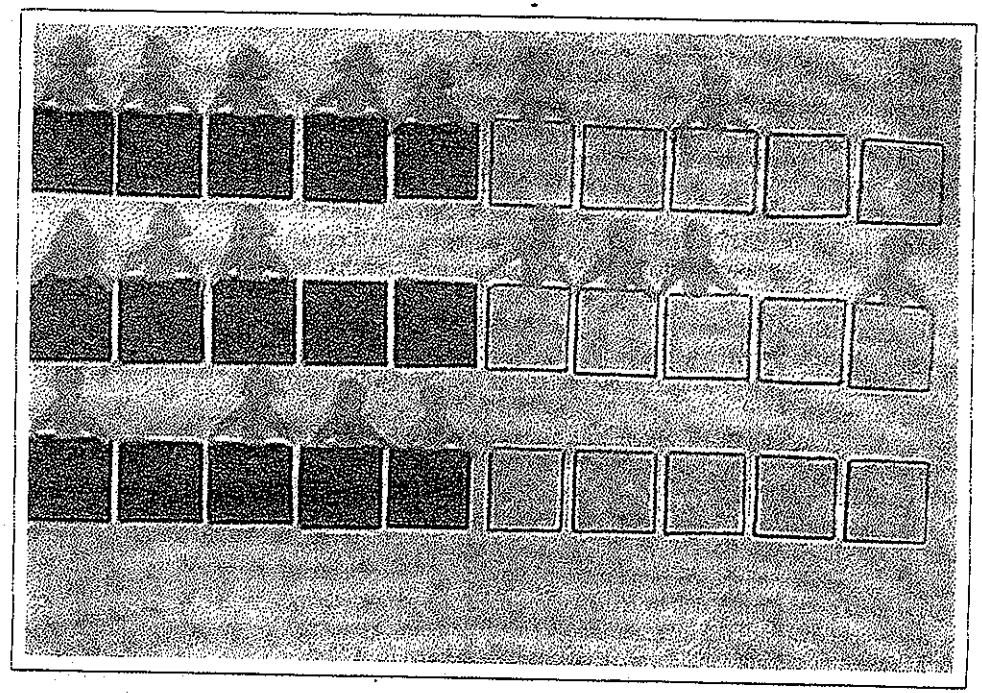
Madeline Chang and Diane Jackson both turned their attendance charts into large arithmetic racks. They made pockets that hung on a clothesline and thus could be moved. Five pockets on each line were blue; five were red. Diane had a class of eighteen preschoolers, so she used two lines of ten pockets. Madeline had a class of twenty-eight children, so she used three rows of ten. Each line of ten was broken into two groups of fives. Children were asked to place a paper doll with their name on it in a pocket on any row when they came into class in the morning. During the morning meeting, they discussed how many children were in school. Let's listen in as Madeline and her students discuss that day's attendance (Figure 6.5 shows the attendance chart). Madeline begins.

"So let's all look at the attendance chart. How many kids are here today? Felicia?"

"Twenty-seven, because there are fifteen here [*pointing to the three groups of five blue pockets*] and then I went sixteen, seventeen, eighteen [. . .] twenty-seven [*counting on to twenty-seven, pointing to each red pocket*]." (See Figure 6.6a.)

"Oh, you didn't start counting from one, you counted on from fifteen," Madeline paraphrases Felicia. Then she asks other children to do the same to check their understanding. "Who knows what Felicia did? Sally?" Sally paraphrases and Madeline continues, "Yes, isn't that neat? Felicia saved herself a lot of counting! Did anyone do it a different way? Sadie?"

Sadie goes to the chart and moves one white pocket over to the left on the top row. In the second row, she moves over two white pockets, and in the third she also moves over two. She explains, "Now I have twenty, because I made another five here." She points to the white pockets that she moved on each line—one, two, and two. "There are seven over there." She points to the white pockets remaining to the right of the ones moved—three, three, and one. "Twenty and seven are twenty-seven," she concludes with certainty. (See Figure 6.6b.)



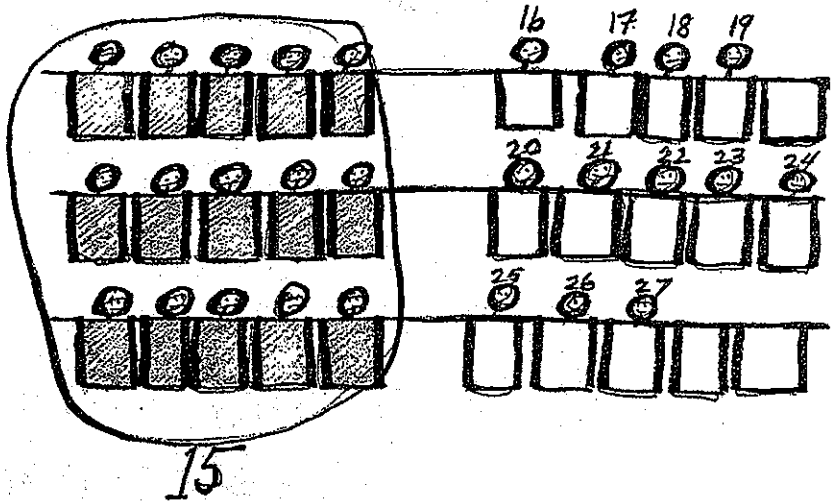


FIGURE 6.6a *Felicia's strategy*

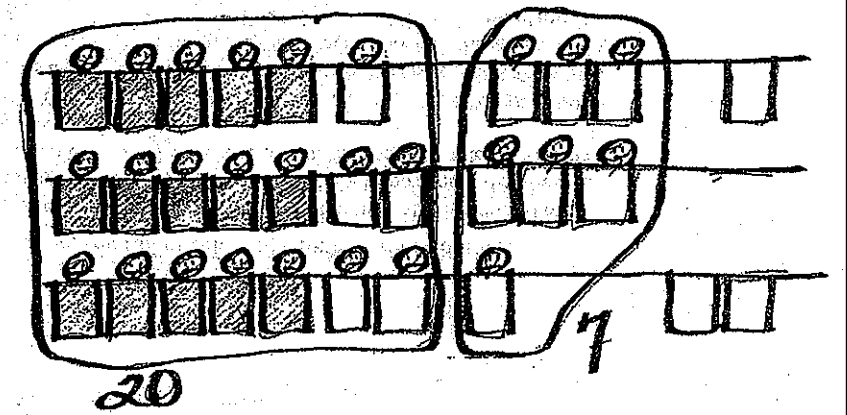


FIGURE 6.6b *Sadie's strategy*

FIGURE 6.6 *Attendance strategies*

"No," says Albert. "I disagree. I got twenty-eight."

"Show us, Albert."

Albert shifts one doll from the bottom to the top, making two groups of ten. "See, I made twenty, too. There's eight on the bottom line . . . oh no, I made a mistake. It is twenty-seven." As Albert double-checks his counting on the bottom line, he corrects himself. He does not make use of the five and two, but instead counts every doll on the bottom row one by one.

In this excerpt, we see how the children group and regroup flexibly, making use of fives and tens; yet some still ignore the structure of the material and try to count one by one. Over time, after using the arithmetic rack in many contexts, however, children journey toward the horizon of automatized facts. Strategies do not change overnight. This is hard work. But as these young mathematicians work, they are grappling with big ideas; they are developing efficient strategies, and they are constructing mental maps of number relationships.

Reprinted from *Games for Early Number Sense: A Yearlong Resource*. Fosnot & Cameron (2007).

Passenger Pairs

The purpose of Passenger Pairs is to encourage children to examine different ways the same number of passengers can be seated on a double-decker bus, and to explore how some groupings are easier than others to recognize on the arithmetic rack because of the five- and ten-structures.

Directions for Playing

The deck (Appendix M) includes 24 cards with different groupings of bus passengers. The children play in pairs. They set up the game by mixing the cards and placing all 24 of them, face up, in four rows of six. They take turns finding two cards that depict the same number. Player One picks up two matching cards, states the number they show and how he knows they are equivalent—moving beads on the arithmetic rack to justify equivalence—and removes the cards from the array. Then Player Two takes a turn selecting two matching cards. In all cases, the partners must agree whether the cards match. The matching pairs are placed in a discard pile or turned facedown, encouraging collaborative rather than competitive play.

Noting the Mathematical Landscape: Openings and Possibilities

Listen in on the players' conversations. Ask children how they know how many bus passengers there are. Some children may find the number quickly but need help articulating their strategies. Note how they use the arithmetic rack. Do they easily slide beads and make use of the five- and ten-structures of the apparatus, or do they count the beads by ones? Do they use compensation easily, for example matching 5 on the top and 2 on the bottom with 4 on the top and 3 on the bottom? Do they make use of the commutative property as they search for pairs?

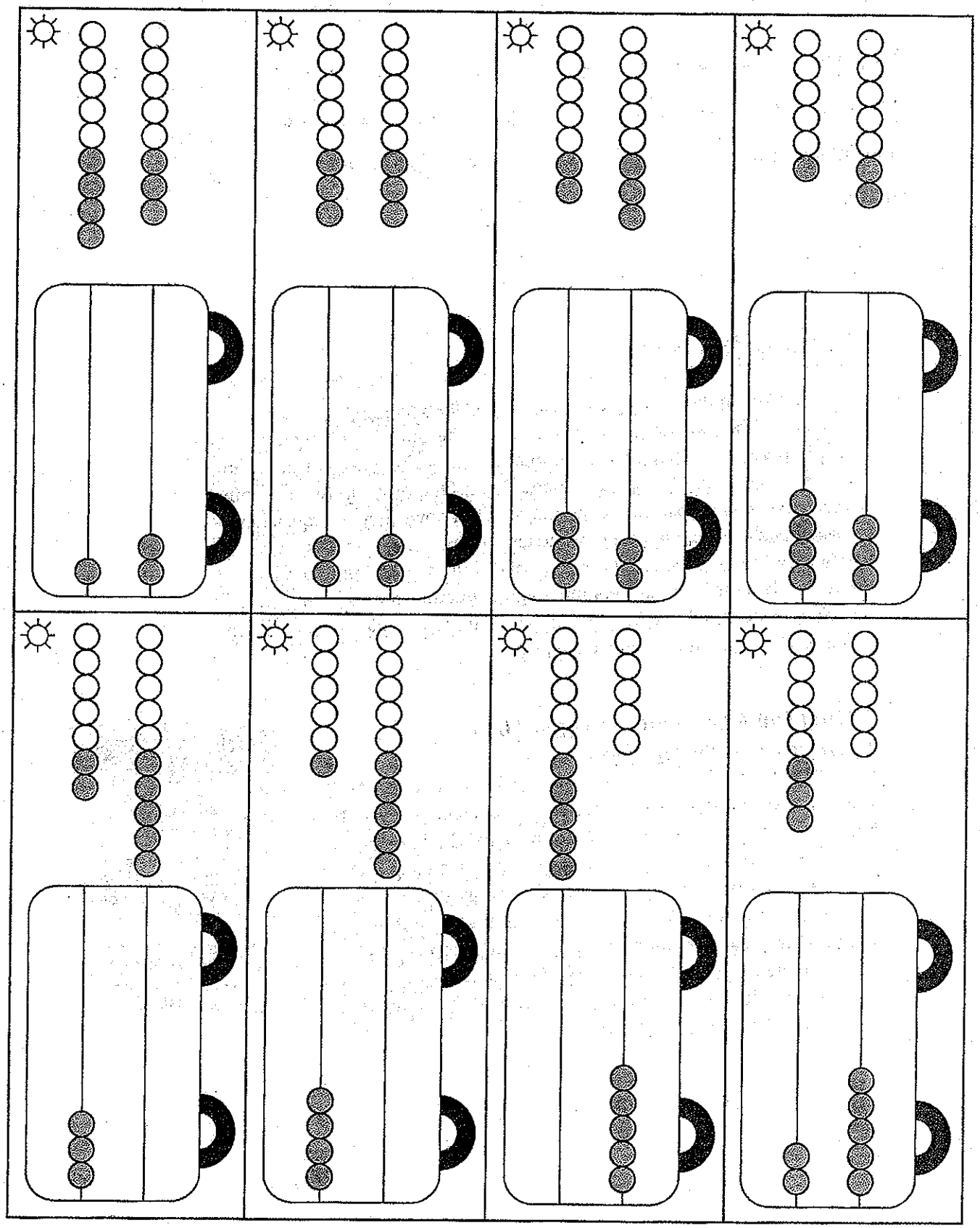
Materials Needed

Arithmetic rack—one per pair of children

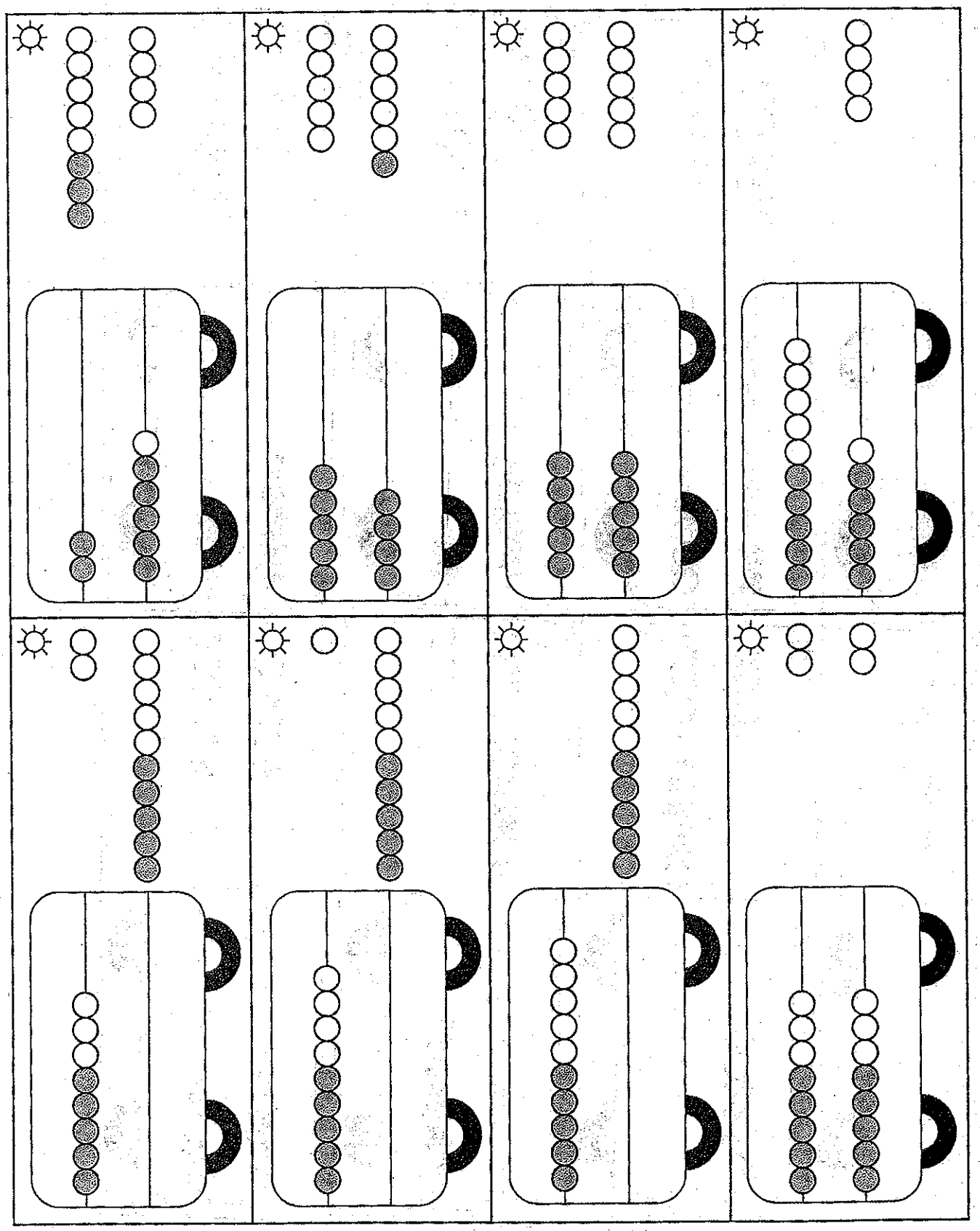
Instructions for making arithmetic racks can be found in Appendix H.

Passenger Pairs game cards (Appendix M)—one set per pair of children

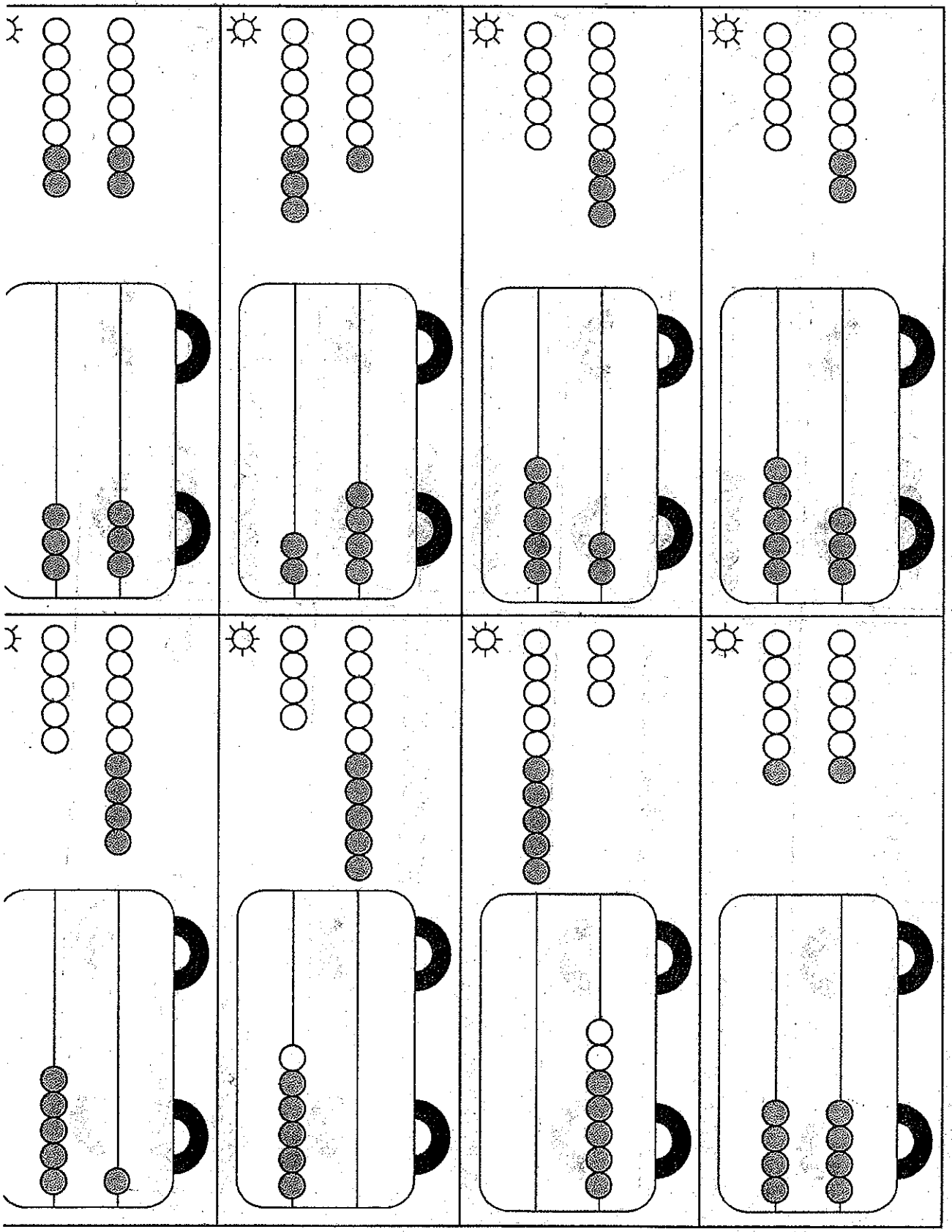
Passenger Pairs game cards



Passenger Pairs game cards



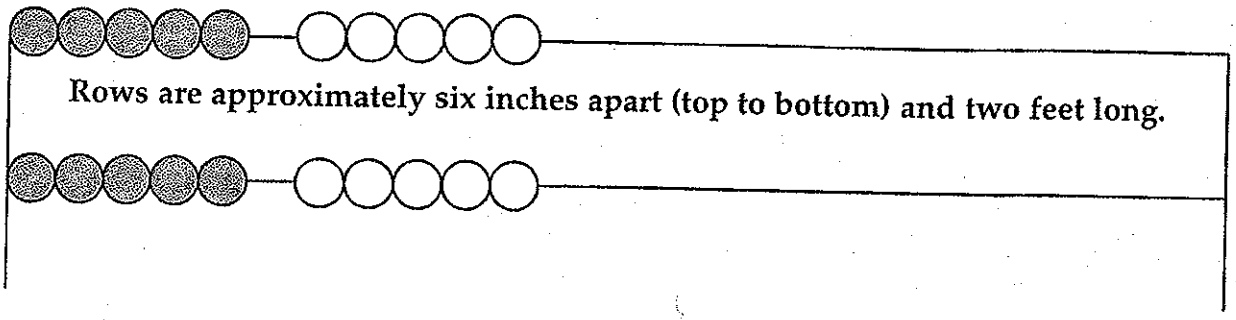
Passenger Pairs game cards



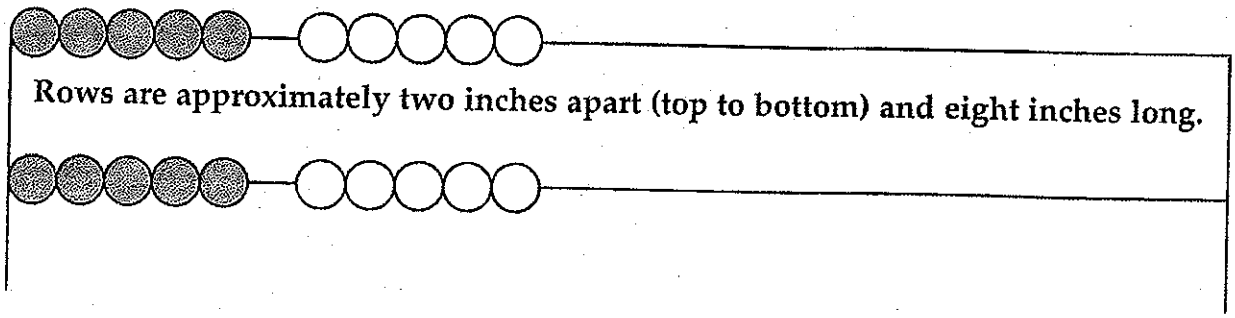
Appendix H

Instructions for making arithmetic racks

To make a class-size arithmetic rack, find a large piece of cardboard, about three feet by one and a half feet. Punch four holes, two on each side, through the cardboard, approximately six inches apart top to bottom, but two feet across. Using wire or thin rope (such as clothesline), string twenty beads in two rows of ten each (five of each color) as in the illustration below. Thread the wire or rope through the holes and twist or tie in the back. [**Note:** If you use wire, it is possible to use connecting cubes in place of beads]



Individual arithmetic racks can be made in a similar fashion to the class-size arithmetic rack. Just use a smaller piece of cardboard, about four inches by twelve inches. Use shoelaces in place of wire and make sure that the beads you use are small enough to move easily along the shoelaces.



For information on where to purchase arithmetic racks, visit www.contextsforlearning.com.

Thinking About the BIG IDEAS of Number Composition

BIG IDEAS	Notes (examples, models, skills or other musings)
A quantity (whole) can be "broken apart" (decomposed) into parts, and the parts can be combined (composed) to form the whole. • For any given quantity (whole), there are different ways to compose and decompose it.	
As numbers get larger, using doubles, 5s, 10s and 100s allow for efficient composition and decomposition of numbers for operations.	

Common Core Standards for Mathematical Practices (at all levels)
How are these practices enacted in classrooms?

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.