erikson
early mathematics education
INNOVATIONS

Learning Lab #2

Afternoon Handouts
Pre-K, K & 1st

Reprinted from *Reasoning Algebraically About Operations* by D. Schifter, V. Bastable, S.J. Russell & S. Monk (Dale Seymour Publications, 2008).

CASE 11

Does order matter when counting?

Lola

KINDERGARTEN, JANUARY

In the Counting Jar this week there were eight checkers—three red and five black. The children "visited" the Counting Jar at various times during the week and counted the checkers. As usual, they recorded their counts on index cards using pictures, numbers, words, or a combination of these. Then they created another set of objects with the same number of items as there were checkers in the Counting Jar.

I did not spend a lot of time at the Counting Jar this week. We had several other math activities that were new, and the children are used to the routine of the Counting Jar. I looked over their index cards and spot-checked a few of their collections. Everyone seemed to have gotten eight for their count. Only two students, Jannah and someone who neglected to write his/her name, indicated anything about the checkers' colors on their cards. Jannah showed three reds and five blacks, and the other person showed four reds and four blacks.

At the end of one math period, we met on the rug to discuss the Counting Jar. Alejandro counted the checkers for us. At first he started to count them any old way, and then he stopped and sorted them by color. He counted the reds first and then the blacks: 1, 2, 3...4, 5, 6, 7, 8. We reiterated that there were eight checkers and determined that there were three red checkers and five black checkers.

Then I posed a big question and told the class I wanted them to really think about it for a minute. "If someone else were to count the checkers and count the blacks first and then the reds, would the total be the same?"

There was, as there fairly often is, a feeling of many students having an immediate response, either verbal or nonverbal. Many of the immediate responses seemed to be affirmative, but I definitely heard at least one or two students say no. As usual, when I scanned the class, there were students who looked a bit unsure, too.

Marisa was one student whose nonverbal response made it clear that she was sure there would still be eight checkers. I asked her about it and she said, "I think you will [still end up with eight], but instead of counting the three first, you'd count the five first."

Tom also thought you would get eight but said he didn't know why. Danielle said if you put the red first and then the black or the black first and then the red, they'd be the same. I asked why, and she said, "If you

took one away it would change it, but if you didn't, it would be the same." Several children then demonstrated taking one checker, or one of each color, away and counted to show that the number of checkers changed. Although nobody commented about adding a checker to change the total, I think some children were having the thought that if you don't actually change the total by taking some away or adding some in, you will have the same amount regardless of what color is counted first.

Emma said that she counted the blacks first and got nine. I asked if she remembered Alejandro's count when he counted the reds first, and she did. I asked if she thought that if you count them in the other order, it could be a different number, and she said she wasn't sure. She was the only one to actually express this uncertainty, although it is likely that other students were also unsure.

Jake said it would be eight whether you counted the reds first or the blacks first. I asked why, and he counted the set both ways, red first and then black first, and got eight both times. "See?" His demonstration applied to this very set of objects.

I posed a new question to the class: Is getting the same total no matter which order you counted something special about the checkers, or would it work with anything? Students who murmured or nodded immediate responses thought it would work with anything. I'm not sure if any of the silent, still students were thinking something different.

Amber said, "If you take the same number, it would be the same." Amber demonstrated her idea with yellow and blue teddy bear counters. I helped her put out 3 yellow teddies and 5 blue teddies. Amber counted the 3 yellows first and then the 5 blues and got 8. I asked Craig, "What if you count the blues first?" and he said, "Eight," without hesitation. I asked how he knew that, and he said his mother taught him. Ariel demonstrated another way to count that still yielded eight: yellow, blue, yellow, blue, yellow, blue, blue, blue.

I asked if this was something special about the number eight, or if it would work with other numbers. No students commented on this. We did try counting a set of 3 blue and 3 green color tiles. The class as a whole seemed bored when we got 6 no matter which order we counted. This conversation was getting way too long for many of the kindergartners, but I really pushed on WHY? WHY? WHY? Marisa said, "It doesn't matter how you do the colors, it matters how many there are."

So, it seems that some students have some understanding of "order not mattering" when counting a set of things that can be thought of as two subsets. It is difficult for them to articulate why, and many of their statements apply to the example at hand. And, as always, I am unsure about students who aren't saying anything, as well as students who nod along once they get a feeling for the group consensus.

I decided to speak to some students one-on-one, specifically the seven students who didn't say anything in the whole-group discussion. Tony counted the whole set each time. Blues first, count them all by ones; reds first, count them all by ones. Lisette, however, laughed when I switched the order of her two color groups. She is a quiet girl whose first language is not English. But she laughed and said, "No change!"

C•8 *Cover All* Game

Objective To provide practice with counting and one-to-one correspondence through a game.

Key Mathematics Concepts and Skills

- Count dots on a die and counters to match the number of dots. [Number and Numeration Goal 2]
- Use one-to-one correspondence to add counters to spaces on a gameboard. [Number and Numeration Goal 2]

Other Skills Cooperation

Terms to Use cover, fill, all, some

Materials Game Masters (*Math Masters*, pp. 52 and 53); dot dice; counters; spinner or numeral dice (optional)

Core Activity

Planning Tip Choose one or more gameboards from *Math Masters*, pages 52 and 53 that seem appropriate for your class. Make several copies of the gameboards and laminate them for durability, if possible. Most children can start with the 9- or 10-square grid. For younger children, you might use the 6-square grid and create dice labeled with dots 1–3 only.

▶ Main Activity

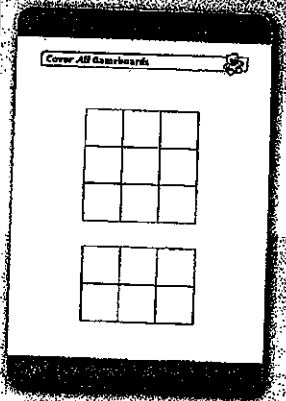
☐ Whole Group ☑ Small Group ☑ Partners ☑ Center

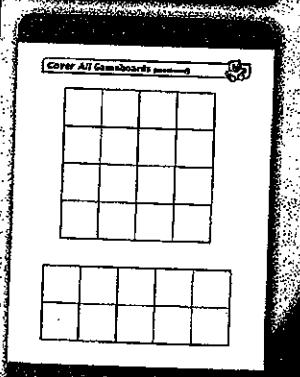
Give each player a *Cover All* gameboard. Players can share a die. Children take turns rolling the die and saying the number of dots. They take that number of counters and place them on their boards, one counter per space. Children continue taking turns adding counters to their boards until one child covers all of his or her spaces. Before playing, children can decide together whether the grid has to be filled by an exact count to end the game. Once children are familiar with the game, place the materials in a Center for children to play with a partner or small group.

You can customize the gameboards by adding stickers or stamps to reflect a current class theme, or simply to make the boards more interesting. Some children might enjoy using pencils or markers to "X" out the squares instead of using counters. For the latter variation, you'll need to provide an ample supply of grids.

Ongoing Assessment: Kid Watching

You can use this activity to observe whether children can count dots on a dice and/or whether they are beginning to recognize dot patterns without counting. You can also informally assess their abilities to use one-to-one correspondence to count counters and spaces.





Adjusting for Age and Development

Later in the year, you can incorporate numeral recognition into the game by using spinners or dice with numerals, instead of dot dice, to determine how many counters to add to the board. You can also increase the challenge as the year progresses with questions such as: *How many more counters do you need to fill your grid? Which grid has more (fewer) counters right now?*

▶ Connections

Snack Connection Children can practice one-to-one correspondence while they help you prepare a snack. Line up small cups or cupcake liners in rows and have children put one each of various items (raisins, crackers, pretzels) into each container.

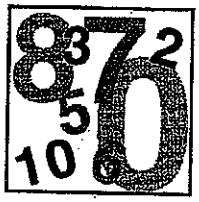
Art Connection Make copies of blank *Cover All* gameboards and invite children to color the squares in a pattern or design.

Game Connection Invite children to invent other games they might play on the *Cover All* gameboards.

Family Connection You might create game bags with materials and directions for *Cover All* that children can check out to play at home with their families.

Lesson 2 We All Count!

KINDERGARTEN



The book *Ten Black Dots* by Donald Crews provides a setting for children to focus on numbers one to ten as they observe how the author used sets of 1 to 10 black dots in pictures of everyday objects. Each child then draws a picture using some number of dots to contribute to a class book.

Key Content

- Translating between dot patterns, pictures, and numbers.
- Sorting and ordering sets.
- Connecting mathematics to real-world situations: using children's literature.
- Developing oral counting skills.
- Investigating patterns in a number sequence.
- Developing number sense for numbers to 100.

ACTIVITY 1

Our Number Book

Materials

For the Student

1–10 self-adhesive dots
construction paper

For the Teacher

Ten Black Dots by Donald Crews
Observational Assessment Record Blackline Masters, Pages 201–202

Teaching the Activity

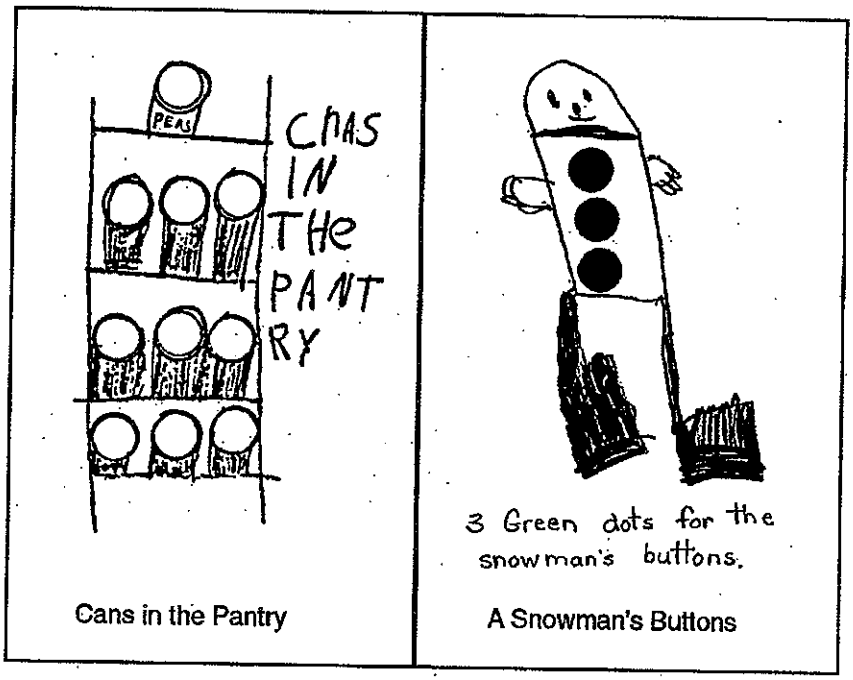
Read the book *Ten Black Dots* by Donald Crews and have children inspect the way the author incorporated some number of dots into each picture. Explain that each child will select the number of dots to use on the page that he or she makes to include in the class book.

To start children thinking, have them brainstorm possible ways to use dots in pictures as you record their ideas. See Figure 7.

- 6 puddles after a rainstorm
- 8 lollipops
- 4 dips of ice cream on one cone
- 3 ice cream cones
- 10 dots on a clown's suit

Figure 7: Dot picture ideas

Then say, *"Think for a minute about what you are going to draw and the number of dots you will need."* Have the child count out the number of dots he or she is requesting. When the pictures are complete, have each child dictate a short description. Then each child can show his or her picture, telling the number of dots used. Have children sort and order the pictures by the number of dots on the page. Discuss how some children used the same number of dots but created different pictures. You can display the pictures on a bulletin board and then make them into a book for the class to enjoy. You may want to send the book home for children to share with their families. See Figure 8.



Cans in the Pantry

A Snowman's Buttons

Figure 8: Sample pages for our number books

ACTIVITY 2

Oral Counting–Buzz!

Children play a circle game to practice a wide variety of oral counting skills.

Teaching the Activity

The game *Buzz!* has been around for a long time. The object of the most common version is for players to count to 100 substituting the word *Buzz!* for the number seven or its multiples without making any mistakes. These kindergarten versions are much simpler; however, careful listening and planning are required to say the correct number in the sequence. The suggested variations of the traditional game offer multiple opportunities for children to practice counting sequences orally. See Figure 9. Introduce these variations one at a time and only after students have practiced easier versions.

The basic procedure of the game remains the same for each variation, allowing children to focus on the counting sequence. Players stand in a circle and count to the designated number in sequential order, one number per student, replacing the target number(s) with the word *Buzz!* When the target number comes up, that student says *Buzz!* and sits down. The game continues in this way, until there is one person standing.

Children participate in this game at many different levels, based on their individual facility with the counting sequence. For instance, children who still need practice with the basic sequence of numbers will probably concentrate on the number that is said immediately before their turn, while children who are very comfortable with the number sequence might be planning their next number five or six players before their turn.

This game can be introduced early in the year and used about once a week. When children are really comfortable with the procedure, this game can be successfully carried out by a substitute teacher if needed.

The teacher can also *Buzz!-Buzz!* children, requiring them to sit down. Some situations that warrant a *Buzz!-Buzz!*:

- saying a number out of sequence. When this happens, the next person in line must say the correct number and play continues.
- waiting an unreasonable amount of time before saying the next number (determine based on your knowledge of a child's skills).
- disrupting the concentration of others by making noises or leaving the circle.

As *Buzz!* is implemented, additional variations will emerge; for instance, students may be ready for skip counting. The whole idea is to start out simple and introduce changes as students are ready for a new counting challenge.

Buzz! Target Number	Counting Sequence	Counting sounds like:
5	to 5	1, 2, 3, 4, Buzz!, 1, 2, 3, 4, Buzz!, 1, . . .
5	to 10	1, 2, 3, 4, Buzz!, 6, 7, 8, 9, 10, 1, 2, 3, 4, Buzz!, 6, 7, 8, 9, 10, 1, . . .
10	to 10	1, 2, 3, 4, 5, 6, 7, 8, 9, Buzz!, 1, 2, 3, 4, 5, 6, 7, 8, 9, Buzz!, 1, . . .
10	to 20	1, 2, 3, 4, 5, 6, 7, 8, 9, Buzz!, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 1, . . .
numbers ending in zero	to 20 to 30 to 40	1, 2, 3, 4, 5, 6, 7, 8, 9, Buzz!, 11, 12, 13, 14, 15, 16, 17, 18, 19, Buzz!, 21, 22, 23, 24, 25, 26, 27, 28, 29, Buzz!, 31, 32, 33, 34, 35, 36, 37, 38, 39, Buzz!, . . .
1	back from 5	5, 4, 3, 2, Buzz!, 5, 4, 3, 2, Buzz!, 5, . . .
2	back from 5	5, 4, 3, Buzz!, 1, 5, 4, 3, Buzz!, 1, 5, . . .
numbers ending in five	to 40	1, 2, 3, 4, Buzz!, 6, 7, 8, 9, 10, 11, 12, 13, 14, Buzz!, 16, 17, 18, 19, 20, 21, 22, 23, 24, Buzz!, 26, 27, 28, 29, 30, 31, 32, 33, 34, Buzz!, 36, 37, 38, 39, 40, 1, . . .
5	back from 10	10, 9, 8, 7, 6, Buzz!, 4, 3, 2, 1, 10, 9, 8, 7, 6, Buzz!, 4, 3, 2, 1, 10, . . .
3	back from 10	10, 9, 8, 7, 6, 5, 4, Buzz!, 2, 1, 10, 9, 8, 7, 6, 5, 4, Buzz!, 2, 1, 10, . . .
numbers ending in zero	to 60 to 80 to 100	1,...51, 52, 53, 54, 55, 56, 57, 58, 59, Buzz!, 61, 62, 63, 64, 65, 66, 67, 68, 69, Buzz!, 71, 72, 73, 74, 75, 76, 77, 78, 79, Buzz!, 81, 82, 83, 84, 85, 86, 87, 88, 89, Buzz!, 91, 92, 93, 94, 95, 96, 97, 98, 99, Buzz!

Figure 9: A chart for *Buzz!* variations

GRADE 1

3·3 Number-Grid Patterns

Objective To guide exploration of skip-counting patterns on the number grid.

1 Teaching the Lesson

Key Activities

Children explore the patterns in counts by 5s, 10s, and 2s on the number grid and in the ones digits of 2-digit numbers.

Key Concepts and Skills

- Count forward by 2s, 5s, and 10s. [Number and Numeration Goal 1]
- Identify the digit in the ones place. [Number and Numeration Goal 3]
- Describe and compare number patterns. [Patterns, Functions, and Algebra Goal 1]

Key Vocabulary

column • row

Ongoing Assessment: Recognizing Student Achievement Use journal page 33. [Number and Numeration Goal 7]

materials

- ☐ *Math Journal 1*, p. 32 and inside back cover
- ☐ Home Link 3·2
- ☐ Transparency (*Math Masters*, p. 311) or a laminated number grid (optional)
- ☐ colored marking pens (optional)
- ☐ slate

See **Advance Preparation**

2 Ongoing Learning & Practice

Children use the language of probability to predict weather.

Children practice and maintain skills through Math Boxes and Home Link activities.

materials

- ☐ *Math Journal 1*, p. 33
- ☐ Home Link Master (*Math Masters*, p. 55)

3 Differentiation Options

READINESS

Children do interrupted skip counting.

ENRICHMENT

Children explore the pattern in counts by 3s on the number grid.

ELL SUPPORT

Children add *row*, *column*, and *diagonal* to their Math Word Banks.

materials

- ☐ Teaching Master (*Math Masters*, p. 56)
- ☐ *Differentiation Handbook*
- ☐ colored pencils
- ☐ "Stop" sign or red paper circle

Additional Information

Advance Preparation For Part 1, use an overhead transparency of a number grid or make a large laminated number grid and use marking pens that are easily erasable. You may wish to find the book *Each Orange Has 8 Slices* by Paul Giganti (Greenwillow Books, 1992) as it relates to lesson content.

Technology

Assessment Management System
Math Boxes, Problem 1
See the **ITLG.**

Getting Started

Mental Math and Reflexes

Children solve problems like the ones below using the number line on the inside back cover of their journals. They record answers on their slates.

Count the hops from

●○○	4 to 10. 6	6 to 11. 5	8 to 13. 5
●●○	9 to 18. 9	5 to 17. 12	4 to 12. 8
●●●	12 to 18. 6	16 to 20. 4	21 to 30. 9

NOTE Circulate as children count on their number lines. Watch for children who include the starting number in their counts—their answers will be 1 more than the correct answer.

Home Link 3•2 Follow-Up

Tally how many children have odd or even numbers of people living at home.

Have children share some of the even and odd numbers they recorded. Write their numbers in two columns on the board—labeled *Even* and *Odd*—as instructed by children. Review the first-grade chant: *2, 4, 6, 8, first graders are really great!* Remind children of the ones digits for even numbers—and perhaps amend the chant to include 0. What are the largest even and odd numbers children wrote?

1 Teaching the Lesson

▶ Exploring Skip-Counting Patterns on a Number Grid

WHOLE-CLASS ACTIVITY

Explain to children that they will be finding patterns on the number grid by marking skip counts with colored dots.

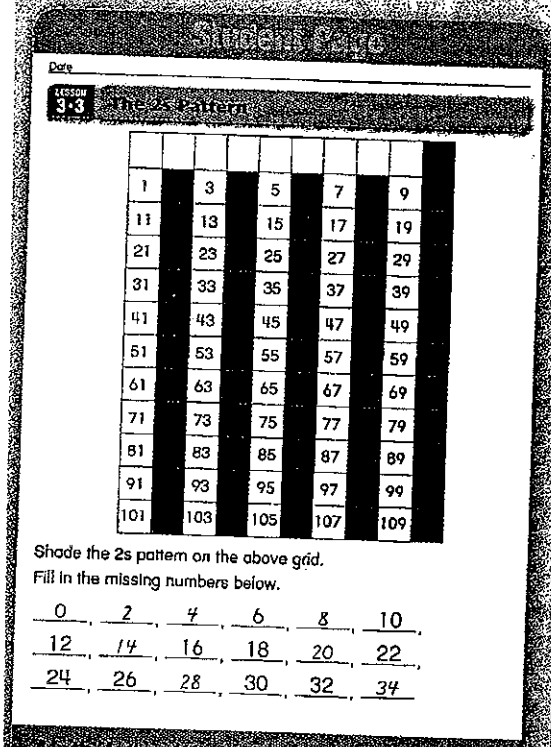
Use either an overhead transparency of a number grid or a laminated number grid. Children count by 5s, one child at a time, in turn. Mark the 5s count (multiples of 5) on the number grid with colored dots. Once a pattern begins to emerge, ask: *How can you find the numbers in the 5s count without actually counting?* The numbers in the 5s and 10s columns.

NOTE Do not expect children to use the word *multiples* at this time.

Adjusting the Activity

It may be helpful for some children to find the 5s counts by counting hops to arrive at each new number.

Make a list of the first few 5s counts and circle their ones digits. Ask children to describe the pattern in the ones digits. The numbers 5 and 0 alternate. *How many numbers does it take for the pattern to be repeated?* 2



Student Page

Date

Lesson 3.3 The 2s Pattern

1		3		5		7		9	
11		13		15		17		19	
21		23		25		27		29	
31		33		35		37		39	
41		43		45		47		49	
51		53		55		57		59	
61		63		65		67		69	
71		73		75		77		79	
81		83		85		87		89	
91		93		95		97		99	
101		103		105		107		109	

Shade the 2s pattern on the above grid.
Fill in the missing numbers below.

0, 2, 4, 6, 8, 10,
12, 14, 16, 18, 20, 22,
24, 26, 28, 30, 32, 34

Math Journal 1, p. 32

Use different-colored dots to repeat this routine with 10s counts. (If you don't have different-colored markers, use different marks, such as dots, checks, and stars, for each set of counts.) Children should observe the following:

- ▷ The 10s are found only in the 10s **column.**
- ▷ All 10s end in 0.

Ask which numbers have been marked more than once. The 10s; this shows that all 10s are in counts by both 5s and 10s.

▶ Exploring the 2s Pattern

SMALL-GROUP ACTIVITY

(*Math Journal 1*, p. 32)

Children work in small groups. Ask all group members to do the following:

1. Make light marks for the counts by 2s on the number grid on page 32 in your journal.
2. Check with other children in your group to see if everyone agrees. Then shade the 2s pattern on your own grid.
3. List the numbers you have shaded at the bottom of the page.
4. Study the number patterns on your grid and talk about what you discovered with your group.

Bring the class together and have volunteers tell about the patterns that their groups discovered.

Possible patterns include the following:

- ▷ The 2s are found in the 2s, 4s, 6s, 8s, and 10s columns.
- ▷ The 2s are all even numbers.
- ▷ The 2s end in the digits 0, 2, 4, 6, and 8.
- ▷ The 2s pattern in the ones digit repeats every five numbers.
- ▷ The pattern repeats in every **row.**

2 Ongoing Learning & Practice

▶ Discussing Weather and Probability

WHOLE-CLASS DISCUSSION

To offer children experience using probability language, ask questions about temperature and weather. Examples include:

- Look outside. Do you think it is *likely* to snow today?
- Is it *possible* or *impossible* that the temperature tomorrow will be warm enough to wear shorts?

▶ Math Boxes 3•3

INDEPENDENT ACTIVITY

(*Math Journal 1*, p. 33)

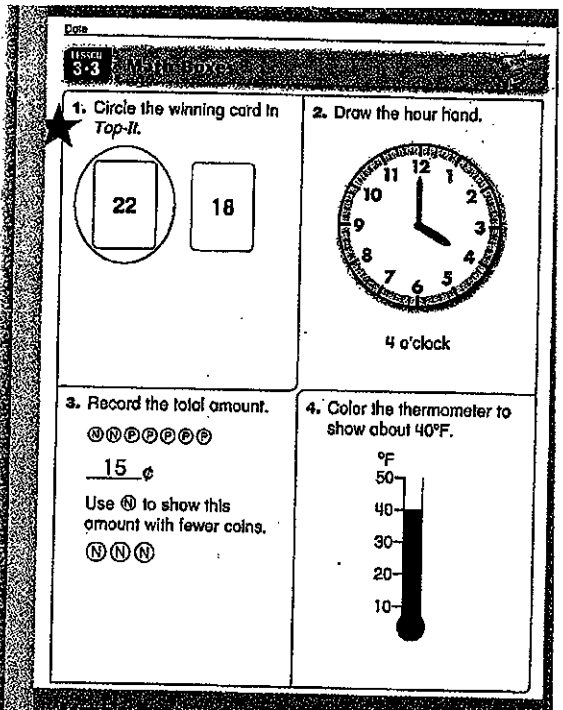
Mixed Practice Math Boxes in this lesson are paired with Math Boxes in Lesson 3-1. The skills in Problem 4 preview Unit 4 skills.

Ongoing Assessment: Recognizing Student Achievement

Math Boxes Problem 1 ★

Use **Math Boxes, Problem 1** to assess children's understanding of comparing numbers. Children are making adequate progress if they circle the greater number.

[Number and Numeration Goal 7]


Date

LESSON 3•3 Math Boxes

1. Circle the winning card in *Top-It*.

22 18

2. Draw the hour hand.

4 o'clock

3. Record the total amount.

(N)(N)(P)(P)(P)(P)

15 ¢

Use (N) to show this amount with fewer coins.

(N)(N)(N)

4. Color the thermometer to show about 40°F.

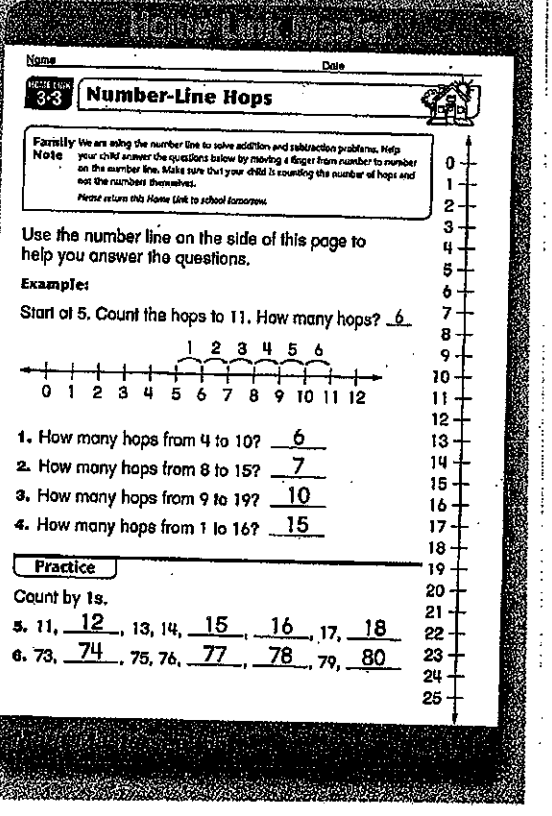
°F 50 40 30 20 10

▶ Home Link 3•3

INDEPENDENT ACTIVITY

(*Math Masters*, p. 55)

Home Connection Children use the number line to find the distance between two numbers. They count the number of hops from one number to another.


Name Date

LESSON 3•3 Number-Line Hops

Family Note We are using the number line to solve addition and subtraction problems. Help your child answer the questions below by moving a finger from number to number on the number line. Make sure that your child is counting the number of hops and not the numbers themselves.

Please return this Home Link to school tomorrow.

Use the number line on the side of this page to help you answer the questions.

Example:

Start at 5. Count the hops to 11. How many hops? 6

1 2 3 4 5 6

0 1 2 3 4 5 6 7 8 9 10 11 12

1. How many hops from 4 to 10? 6
2. How many hops from 8 to 15? 7
3. How many hops from 9 to 19? 10
4. How many hops from 1 to 16? 15

Practice

Count by 1s.

5. 11, 12, 13, 14, 15, 16, 17, 18
6. 73, 74, 75, 76, 77, 78, 79, 80

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

3 Differentiation Options

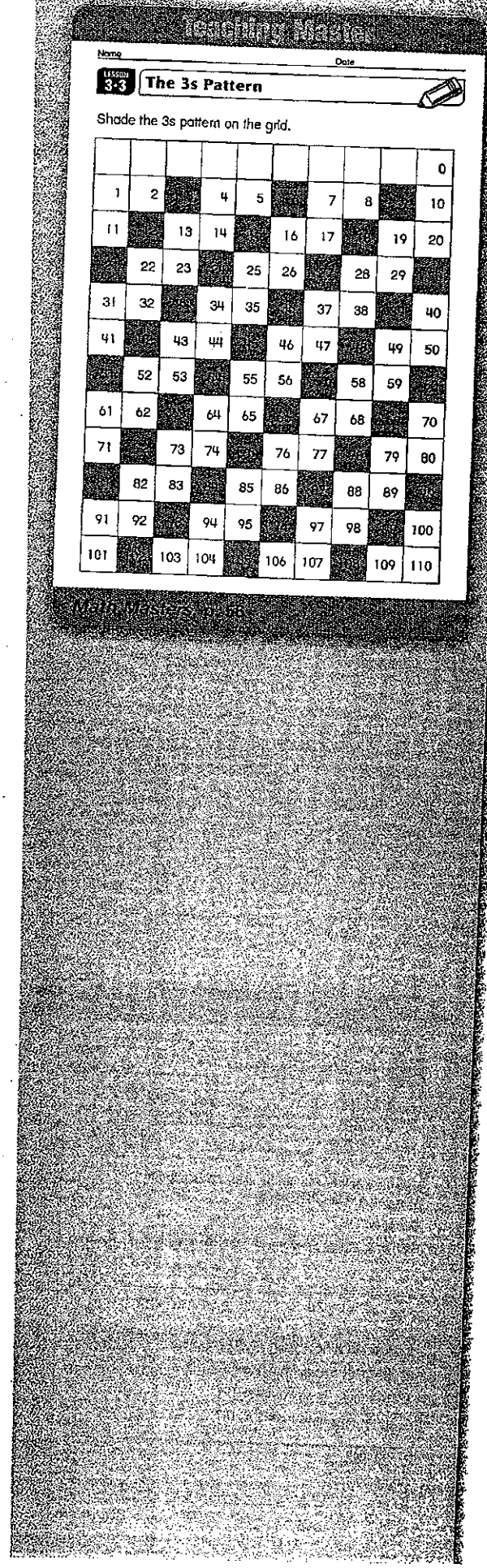
READINESS

SMALL-GROUP ACTIVITY

5–15 Min

▶ Counting with Stops

To review skip counting, have children do interrupted skip counting. Begin by saying a number to a small group of children. Ask them to continue counting on from that number. After they have said a few numbers, hold up the "stop" sign to indicate that children should stop counting. Begin counting again from a higher number. For example: 11, 12, 13, 14, *Stop! Now begin at 19.* 19, 20, 21, ... Repeat the activity, counting by 2s, 5s, and 10s.



Name _____ Date _____

Lesson 3-3 **The 3s Pattern**

Shade the 3s pattern on the grid.

									0
1	2		4	5		7	8		10
11		13	14		16	17		19	20
	22	23		25	26		28	29	
31	32		34	35		37	38		40
41		43	44		46	47		49	50
	52	53		55	56		58	59	
61	62		64	65		67	68		70
71		73	74		76	77		79	80
	82	83		85	86		88	89	
91	92		94	95		97	98		100
101		103	104		106	107		109	110

Math Masters, p. 56

ENRICHMENT

Exploring the 3s Pattern

(*Math Masters*, p. 56)

SMALL-GROUP ACTIVITY

15–30 Min

To further explore number-grid patterns, have children count by 3s and shade the pattern. Ask each child to begin by making light marks for the counts by 3s in the number grid on *Math Masters*, page 56. Then ask children to shade the 3s pattern. Have children study the shaded patterns and discuss what they discovered with their groups. Prompt children to describe the number pattern in the *diagonal* that starts with 9 in the second row and goes down and to the left. Note that this diagonal shows counts by 9s. To support English language learners, write *diagonal* on the board. Ask children to identify some additional diagonals on the number grid.

ELL SUPPORT

Building a Math Word Bank

(*Differentiation Handbook*)

SMALL-GROUP ACTIVITY

5–15 Min

To provide language support for navigating the number grid, use the Word Bank template found in the *Differentiation Handbook*. Ask children to write the terms *row*, *column*, and *diagonal*, draw pictures representing the terms, and write other words that describe them. See the *Differentiation Handbook* for more information. Make a classroom poster to provide further support. (See below for a suggestion.)

ROW

COLUMN

DIAGONAL