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INNOVATIONS

Learning Lab #2

Agenda

- *Begin with Breakfast*
- Math Investigation
- Development of ***Counting, Cardinality & Number Sense*** from Age 3 to Grade 3: the BIG IDEAS
  - Video Analysis
  - Common Core Standards
- High-Impact Strategies for Mathematics
  - ***Learners reiterate/rephrase/articulate other learners' thinking.***
  - Extended response on ISATs
- High-Quality Books to Spark Mathematical Thinking & Action
- *Break for Lunch*
- Developmental Span
  - Case Study Discussion
  - Lesson Analysis

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During Learning Lab # 2, each cohort will agree on norms for our time together. We feel these are critical to creating a community where everyone can engage in learning. Please review the list of possible norms. **Decide on 3 or 4 that you feel would be essential to building a productive learning community**. Feel free to add another if your idea is not represented. Be prepared to discuss why these norms are important to you.

Possible Learning Lab Norms

honor start & end times	be fully engaged
share the floor	respect varying perspectives
confidentiality	participate equally
have a focus (follow agenda)	maintain momentum
reach closure (next steps...)	other ideas?

Thank you for your thoughtfulness!

Focus for Today

- Content: ***Counting, Cardinality & Number Sense***
- Strategy: ***Learners reiterate/rephrase/ articulate other learners' thinking***

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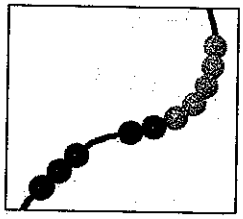
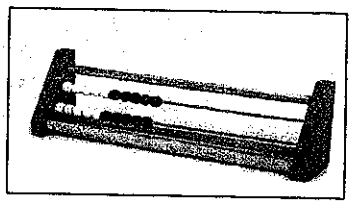
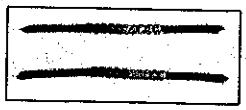
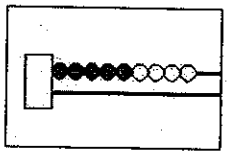
Developing Visual Number Sense Through Models









Counting Frames
(Rekenrek)

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Counting in Context

- Each table group will get a large quantity of red and white beads.
- How many 20-bead (10 red, 10 white) counting racks could be made with these beads? How many of each color are left over?
- How many 10-bead (5 red, 5 white) counting racks could be made with these beads? How many of each color are left over?
- You will make a poster will showing your answers and the process you used to figure it out.

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How do children develop understanding of counting, cardinality & number sense?

What are the ***Big Ideas***?

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Thinking About the BIG IDEAS of Counting, Cardinality & Number Sense

BIG IDEAS	Notes (examples, models, skills or other musings)
There are principles for counting. • ***One-to-one Principle***: Each item in a set is counted once & only once. • ***Stable Order Principle***: Number names must be used in a stable, repeatable order. • ***Cardinal Principle***: The final number used when counting represents the quantity of the set counted. • ***Order Irrelevant Principle***: No matter what order the items in a set are counted, the result is the same. • ***Abstraction Principle***: Counting can be applied to any set, including groups (like dozens) or non-physical entities (like ideas).	
Quantity is an attribute of a set of objects, and we use numbers to name specific quantities.	
As numbers grow larger, we group by tens to create new units. • Because we group by tens, we can represent all numbers using ten digits (0 to 9), and there are patterns to how numbers are represented.	

Number is Complex!

Counting is Complex, too!

A ***Big Idea***

There are principles (rules)
for counting.

One-to-One Principle (Big Idea)

Each item in a set
is counted
once and only once.

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1-to-1 Correspondence Principle: What learning looks like (skills)

One number is named
for each object pointed at.

When objects are
counted in groups,
appropriate skip counting is used.

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Stable Order Principle (Big Idea)

Number names must be said in a stable, repeatable order.

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Stable Order Principle: What learning looks like (skills)

- Mastery of the number name sequence used by culture
- Can count up from given number
- Can count down from given number

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Cardinality Principle (Big Idea)

The final number used
when counting
represents the quantity
of the set counted.

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Cardinality Principle:
What learning looks like (skills)

- When asked, "How many altogether?" names the last number (without re-counting).
- When given a story problem, can model the count, using manipulatives, drawings & words.

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Order Irrelevant Principle (Big Idea)

No matter what order
the items in a set are counted,
the result is the same.

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Order Irrelevance Principle:
What learning looks like (skill)

To assure accuracy of counting,
some system is used
such as lining up,
pushing away
or somehow noting each item
as it is counted.

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Abstraction Principle (Big Idea)

Counting can be applied
to any set,
including groups (like dozens) or
non-physical entities (like ideas).

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Abstraction Principle:
What learning looks like (skill)

- Can name the set that has been counted.
- Can unitize: collect items into same-size groups and then count the groups.

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Why do we Count?
A *Big Idea*

Quantity is an attribute
of a set & we use numbers
to name specific quantities.

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A *Big Idea*

- As numbers grow larger, we group by tens to create new units.
  - Because we group by tens, we can represent all numbers using ten digits (0 to 9), and there are patterns to how numbers are represented.

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How are the **Big Ideas**
& related **skills**
we've been discussing
reflected in
the *Common Core Standards*?

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What happens in an effective mathematics classroom?

- Students & teachers believe & experience that math is a sense-making activity to help us understand our world.
- Students & teachers model math in multiple ways.
- Students & teachers talk about math.
- Students & teachers ***do*** math.

How were these **principles** illustrated in the classroom we saw in the video?

How did the **teacher's choices** influence the **children's construction** of mathematical understanding?

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Using good books to help students explore numerosity & number sense

- Where's the math in the book?
  - What are the Big Ideas?
- How can you bring the math out of the book?
  - What kind of activities could you use to develop children's thinking about the math of the book?

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Common Core Standards Relating to Counting, Cardinality & Number Sense

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Kindergarten	1st Grade	2nd Grade	3rd Grade
Know number names and the count sequence. • Count to 100 by ones and by tens. • Count forward beginning from a given number within the known sequence (instead of having to begin at 1). • Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). Count to tell the number of objects. • Understand the relationship between numbers and quantities; connect counting to cardinality. * When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. * Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. * Understand that each successive number name refers to a quantity that is one larger. • Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. Work with numbers 11–19 to gain foundations for place value. • Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 +8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	Extend the counting sequence. • Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. Understand place value. • Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: * 10 can be thought of as a bundle of ten ones — called a "ten." * The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. * The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).	Understand place value. • Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: * 100 can be thought of as a bundle of ten tens — called a "hundred." * The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). • Count within 1000; skip-count by 5s, 10s, and 100s. • Read and write numbers to 1000 using base-ten numerals, number names, and expanded form. • Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.	Use place value understanding and properties of operations to perform multi-digit arithmetic • Use place value understanding to round whole numbers to the nearest 10 or 100. • Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. • Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations. Develop understanding of fractions as numbers. • Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.

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High Quality Books to Spark Children's Thinking About Counting, Cardinality & Number Sense

Anno, M. *Anno's Counting Book.* HarperCollins Publishing, 1975.

Bang, M. *Ten, Nine, Eight.* Greenwillow Books, 1983.
Tambien en espanol: Bang, M. *Diez, Nueve, Ocho.* Mulberry.

Crews, D. *Ten Black Dots.* Greenwillow Books, 1986.
Tambien en espanol: Crews, D. *Los Diez Puntos Negros.* Greenwillow Books.

Demi. *One grain of rice: A mathematical folktale.* Scholastic, 1997.

Giganti, P. *Each orange had 8 slices: A counting book.* Greenwillow, 1992.

Giganti, P. *How many snails? A counting book.* Greenwillow, 1988

Harshman, M. *Only one.* Scholastic, 1993.

Hoban, T. *26 letter and 99 cents.* Greenwillow Books, 1987.

Hoban, T. *Count and See.* Simon & Schuster, 1972

Hoban, T. *Let's Count.* Greenwillow Books, 1999.

Hoban, T. *More, Fewer, Less.* Greenwillow Books, 1998.

Hong, L.T. *Two of everything.* Albert Whitman & Company, 1993.

Kellogg, S. *How Much is a Million?* Harper Collins Publishing, 1985.

McMillan, B. *Eating fractions.* Scholastic, 1991.

Merriam, E.*12 Ways to Get to 11.* Aladdin Paperbacks, 1993.

Nolan, Helen. *How much, how many, how far, how heavy, how long, how tall is 1000?* Scholastic, 1995.

Princzes, E. *One Hundred Hungry Ants.* Houghton Mifflin Company,1993.

Schlein, Miriam. *More than one.* Scholastic,1996.

Tang, G. *The best of times.* Scholastic, 2002. (ages 7-12)

Tang, G. *The Grapes of Math.* Scholastic, 2001. (ages 3-6)
Tambien en Espanol: Tang,G. *Come Una y Cuenta 20.* Everest

Tang, G. *Math appeal: mind-stretching math riddles..* Scholastic, 2003. (ages 7-12)

Tang, G. *Math fables: lessons that count.* Scholastic, 2004. (ages 3- 6)

Tang, G. *Math for All Seasons.* Scholastic, 2002. (ages 5-8)
Tambien en Espanol: Tang, G. *Un, Dos, Tres, El Ano se Fue.* Everest.

Tang, G. *Math potatoes: mind-stretching brain food.* Scholastic, 2005. (ages 8-13)

Tang, G. *Math-terpieces: the art of problem-solving.* Scholastic, 2003. (ages 5-12)

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Using an Interactive Read-aloud to Spark Mathematical Thinking

- **Book title & author:**
  - **Genre** *(informational picture book? concept book? story/narrative?)*
  - **Type of illustrations** *(photographs? realistic illustrations? playful illustrations?)*
  - **Text density and Features of print** *(Is there a lot, a little or a moderate amount of text? Are there any ways attention is drawn to the text by playing with size or position?)*

- **Where's the Math?** *Which of the big ideas about math could this book be used to illustrate, explain, demonstrate or mathematize? Be specific.*

- **Making Connections:** *What is something you might say as you introduce the book that will direct the children's attention to your mathematical focus?*

- **Creating Conversations:** *Consider both the text and the illustrations as you develop at least three open-ended questions to use to encourage the children to think about the math in the book.*

- **Bringing books to life:** *What kind of activities could you use to extend the book? How will these inspire the children to explore the big ideas about math in the book and other connected mathematical concepts?*