The Use of Concrete Objects in Early Mathematical Learning

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With David Uttal

The use of concrete objects, or manipulatives, to foster mathematical learning is a common practice in many early childhood classrooms. The approach is regarded as developmentally appropriate because it allows young children to learn through hands-on experiences with objects they can hold and manipulate. Though the practice of using manipulatives is virtually universal, some studies show that young children do not make connections between their experiences with manipulatives and the mathematical concepts they are designed to teach. Why don’t children make the connections? Does this mean that teachers should not use manipulatives to help children learn early math?

To explore these questions, we interviewed David Uttal, a professor of psychology and education at Northwestern University. Uttal’s research covers a wide range of topics in cognitive development, including mental representation, spatial reasoning, and early symbolization. In this chapter, we focus on his work related to children’s understanding of symbols and explore what this research suggests about the practice of using manipulatives.

Box 1: What are manipulatives?

Manipulatives take many forms. Some are objects that were designed for specific types of mathematics learning, such as Cuisenaire Rods and Dienes Blocks. Others are materials that are commonly found in classrooms but are used for a variety of purposes, such as Unifix cubes, pattern blocks, and counting bears. Still others are household objects such as paper clips, coins, cereal, and crackers. According to Uttal, regardless of the type, what makes an object a manipulative is the fact that a teacher can use it to help children learn mathematical concepts.

The use of manipulatives is built on theories about young children’s learning and development by developmental theorists such as Friedrich Froebel, Jean Piaget, Jerome Bruner, and Maria Montessori. The theories maintain that young children are concrete thinkers while mathematical concepts are abstract. These theories also assert that young children are active learners; they must construct their own knowledge in order to understand the world. Thus, for children to learn abstract mathematical concepts, the concepts need to be translated into objects that young children can see and touch. This helps them build mental images that support the development of their understanding of abstract ideas.

Background

Uttal’s interest in the role of concrete objects in children’s learning can be traced back to his time as a doctoral student at the University of Michigan. One of his mentors, Harold Stevenson, was one of the first American psychologists who studied Chinese and Japanese school achievement. In his seminar at the University of Michigan, recalled Uttal, Stevenson spoke often about the use of teaching tools and curricular methods in Chinese and Japanese classrooms. In Japanese
classrooms, for example, Stevenson noted that teachers tended to use, again and again, a small manipulative set consisting of a few simple tools such as blocks and other shapes. Children would pull out the kit, apply it to different kinds of math problems, and, in the process, become very familiar with it. Although the tools were not new, children would apply them to new problems. In contrast, American teachers sought variety. They might use Popsicle sticks in one lesson and marbles, Cheerios, M&Ms, checkers, poker chips, or plastic animals in another. To Uttal, Stevenson’s observation was thought-provoking. Does type of manipulative matter? Does variety and novelty help? Years later, Uttal worked with Judy Deloache in her lab at the University of Illinois at Urbana-Champaign and participated in her “scale model” studies that examined the development of symbolic thinking. As he worked on these studies, he remembered his conversations with Stevenson about manipulatives and saw a connection between the two ideas.

Section 1: Scale model task and the dual representation hypothesis

Judy Deloache’s studies used the “scale model” task, which tested whether young children can understand that one object can represent another object. Central to the task are two rooms: a normal-sized room furnished with a sofa, chairs, a table, etc., and an exact replica of the normal-sized room that is similar in size to a dollhouse (see Illustration 1). In the experiment, a child is first introduced to the model room. While the child watches, the experimenter hides a “Small Snoopy” dog under a cushion on the sofa in the model room. Then the experimenter explains that there is also a “Big Snoopy” who lives in a big room that looks just like the small room, and he is hiding at the exact place that Small Snoopy is hiding in the small room. The experimenter takes the child into the actual room and asks her to find “Big Snoopy,” reminding her again that it is in the same place that Small Snoopy was in the small room.

Illustration 1: Scale Model in Judy Deloache’s Experiment

Studying children from 2 to 4 years of age, the researchers found that children younger than 3 almost always failed to locate Big Snoopy in the actual room, finding it on average only about 20% of the time. What do these difficulties mean? Could it be that these children simply forgot where Little Snoopy was hidden in the model room? To check this, the experimenters took children back to look at the model after their search for Big Snoopy. The children readily located the Small Snoopy under the cushion on the small sofa. Their difficulty in locating Big Snoopy was
not due to a problem with memory. Rather, they were not able to use Small Snoopy’s location in the model to help them find Big Snoopy in the actual room.

Deloache and her colleagues explained this phenomenon with the dual representation hypothesis. According to this hypothesis, the reason the 2-year-olds could not locate Big Snoopy was that they did not understand the relationship between the model and the actual room. Specifically, they did not see that the model could be viewed as a symbol that represented the room next door. In other words, the children were incapable of representing a concrete object as both an object in and of itself and as a symbol representing something else.

To test this hypothesis, Deloache and her colleagues conducted another version of the study in which children were introduced to a “special machine” that could “shrink” large rooms (see Illustration 2). The child watched as the experimenter hid a large troll doll in a large portable tent. Then the child left the room, and the experimenter pretended to turn on the machine. While the machine made a noise, the child was told that the room was shrinking. When they entered the room again, in place of the tent was an exact replica of the room, only smaller. This time the children were much more successful in locating the doll than they were in the Snoopy task. Why were they successful this time? The researchers argued that since children thought the small model and the large tent were the same object, they did not need to see the model as both an object and a symbol. In other words, they did not need to use dual representation to solve the problem.

Illustration 2: Shrinking Room Experiment

Section 2: Manipulatives as symbols of math concepts

What does the scale model task have to do with manipulatives? When used to support the acquisition of mathematical concepts, a manipulative is like the model of the room: it is both a concrete object and a symbol that represents something else—a mathematical idea. When Uttal observed children’s difficulty connecting the model to the actual room in the scale model task, he wondered if they might have difficulty connecting manipulatives to the math concepts they are designed to represent. As Uttal explains it, in order for manipulatives to help children learn, children must understand that the manipulatives represent a concept in mathematics. He and his colleagues subsequently wrote several papers exploring these ideas. In these papers, they brought
to light a new perspective about concrete versus abstract learning, and about the kind of thinking required to learn using manipulatives.

The researchers also called into question the idea of using manipulatives in spontaneous, self-directed play. Teachers frequently use manipulatives as a self-guided tool for mathematical learning, but it is not clear that children will understand the relation between the manipulatives and the math concept they represent during this kind of play. After all, to quote the math educator Deborah Ball, “understanding does not travel through the fingertips and up the arm…. Mathematical ideas really do not reside in cardboard and plastic materials” (p. 47). Uttal and his colleagues argued that it is unlikely that young children can directly perceive the relation between manipulatives and abstract mathematical concepts and symbols. Rather, their comprehension of these relations needs to be guided by teachers’ instruction.

**Box 2: Do manipulatives work?**

Many researchers have looked at whether manipulatives help children learn the mathematical concepts they are intended to teach. It is difficult to draw a general conclusion about whether they work overall, because there are so many different contexts and ways to use manipulatives. Recently, researcher Kira Carbonneau and her colleagues conducted a meta-analysis on the topic, examining 55 studies that compared outcomes when students were taught with or without manipulatives. Some of these studies found that children performed better with manipulatives, whereas others showed that they performed about the same as children who did not use manipulatives. A few found that children actually performed worse with manipulatives than without them. Overall, Carbonneau and her colleagues found that manipulatives seemed to help students to a small to moderate degree, but that the effects were influenced by several factors, including math content, amount of accompanying instruction, and age of the students. With regard to age, they found that the instruction with manipulatives was least effective for children between the ages of 3 and 6 years compared to older age groups (7- to 11-year-olds and 12 years and older).

**Section 3: Factors that affect children’s ability to use the model as a symbol**

Deloache, Uttal, and their colleagues conducted further experiments with the scale model task in order to find out whether any particular factors affect children’s ability to achieve dual representation, or to be able to view the model as a symbol of the larger room. In one experiment, children were allowed to play with the room model for 5 to 10 minutes before they were asked to locate Big Snoopy in the actual room. These children were less successful in locating Big Snoopy than children who did not play with the model. Other children saw only a picture of the room model, and still others saw the actual model but were not able to touch it because it was placed behind glass. Both of these conditions improved children’s performance on the task.

Why would more time with the model in the above experiment matter, and why would contact with an actual model versus a picture, or viewing a model behind glass matter? The researchers explained that playing with the actual model beforehand led children to focus on it as an interesting toy. They consequently had more difficulty thinking of the model as a representation of the room. When the model was shown as a picture or was not accessible to play with, they were more likely to be able to view it as a symbol of the actual room.
In another variation of the study, researchers modified the instructions children were given about the relationship between the room model and the actual room. In one condition the experimenters proceeded as they did in the usual version of the study, calling children’s attention to the detailed similarities between the model and actual room. For example, they brought the small couch into the actual room and said, “Look—this is Big Snoopy’s big couch, and this is Little Snoopy’s little couch. They’re just the same.” In the other condition, they omitted this part of the instructions. Children were less successful at finding Big Snoopy when the similarities between the rooms were not made explicit; these children succeeded at chance level (i.e., at the same rate as if they were just guessing). Thus, without instructions, children were not able to see how the model room related to the room next door. They needed an adult to show and tell them that the model represented the actual room.

Box 3: Concrete and abstract thinking in mathematics

As described in Box 1, many theorists have argued that children learn through concrete experiences. To learn mathematics, though, children must grasp ideas that are abstract. Even basic math concepts, such as number, are abstract. For example, when a teacher is helping young children learn the concept of four, she may place four paper cups on the table. Cups are concrete objects that children know can be used to drink milk, juice, or water. However, in this situation, the function of the cups, as well as the particular shape, size, or color, does not matter. What matters is the quantity that the objects represent: the cardinal number four (for an explanation of cardinal numbers, see Chapter 1 (Levine), Box 2). Four is an abstract concept: the “fourness” of the four cups has no direct relation to the objects—they could be cups or spoons, red or blue.

Uttal asserts that mathematics at its core is an abstract system. Learning the meaning of the word “four” (and eventually the symbol “4”) allows children to reason about numerical relations independent of any physical representation of the concepts. For example, as he explains in one paper, one can figure out the solution to the question, “What is 1 more than 4?” without thinking, “4 of what?” or “1 more of what?” So, to be able to engage in mathematical thinking, he concludes, children must acquire a system that is “distinctly not concrete.” The transition from concrete thinking (focusing on the actual cups) to abstract thinking (considering the quantity four, and figuring out what one more than 4 is), as well as whether this is indeed the necessary direction of learning, is the subject of many of Uttal’s papers.

Section 4: Perceptually rich manipulatives in early math learning

In recent years, Uttal and his colleagues investigated the idea of whether making manipulatives more attractive affects children’s gains from them. They looked specifically at a manipulative system called Digi-Blocks. The system includes blocks representing ones, tens, hundreds, etc., where ten “one” blocks fit inside a “ten” block, and ten “tens” fit inside a “one hundred” block. Digi-Blocks are supposed to help children understand place value, including arithmetic problems that require “carryover” or “borrowing.” For example, when solving 31 – 22, a child must figure out how to take away the 2 from 1 in the units column. With the Digi-Blocks, they can take a “ten” block, open it to reveal 10 “one” blocks, and then use the 11 “ones” to subtract 2 (see Illustration 3).

The researchers had previously found that children who were instructed with Digi-Blocks learned to solve “carryover” problems using the blocks, although they could not immediately transfer this knowledge to an equivalent problem in written form. In this experiment, the
researchers explored whether making the manipulatives perceptually attractive affected children’s ability to use the blocks to solve problems. Two groups of children who had just finished the first grade participated in the experiment. One group was assigned to use the standard manipulatives while the other used a distinctive set that was decorated with different colors and unique designs such as swirls and polka dots. Children were asked to use the manipulatives to solve two-digit subtraction problems that required the “borrowing” or “carryover” method for solution.

**Illustration 3: Digi-Blocks System**

The results showed that although not statistically significant, the standard manipulatives group performed better than the distinctive manipulatives group. Children in the distinctive group were more likely to play with the blocks (e.g., build towers) than use them to solve the problems. Furthermore, children in the distinctive group were less likely to use the proper procedures and therefore more likely to make errors in calculation than those in the standard group. Based on the results, Uttal and colleagues concluded that, “visually distinctive, compared with standard, manipulatives tended to detract from mathematics instruction.” (p. 6.)

**Section 5: Challenges in studying the efficacy of teaching early math with manipulatives**

Although he writes extensively about mathematical learning with manipulatives, Uttal is keenly aware of the limitations of the scale model studies and other developmental psychology research he conducted for having clear implications for classroom practices. The scale model studies, for example, were not designed to directly examine the relationship between manipulatives and mathematical learning. The experiments took place in well-designed, highly-controlled laboratory environments, where the emphasis is on rigorous control of subjects, materials, and testing procedures. In classrooms, however, variations in procedures are the norm, and material is adapted to be relevant to the context. In fact, the more Uttal explored the relationship between manipulatives and math learning, the more he realized both the significance and the complexity of the issue. In his words, “The problem is too rich and too important to be based on just laboratory studies.”

Recognizing the constraints of the laboratory study, Uttal attempted to work with classroom teachers to explore how manipulatives affect math learning and teaching. He was confronted with a number of challenges. For example, to study the effectiveness of manipulatives in math learning, he would need to compare two groups of classrooms, where one group would use...
manipulatives for math learning and the other would not. However, for most teachers, particularly those working in early childhood classrooms, the idea of giving up the use of manipulatives would be unthinkable. Further, to gain a deep understanding of the impact of manipulatives on math learning, he would need to investigate the issue under different circumstances, varying type of manipulative, frequency of use, and details of associated instruction—all factors that would require teacher collaboration. However, while many teachers are interested in exploring different methods of teaching and learning, they are often cautious about participating in studies that require a commitment of instructional time but that are not guaranteed to have a positive impact on students’ performance.

Uttal said that over the years he has gained a more nuanced understanding of the role of manipulatives in children’s learning by observing classrooms and conversing with teachers. For example, he recalled, a kindergarten teacher once told him about children playing with pretend spaghetti using either yarn or real-looking plastic spaghetti. The teacher noticed that if children first used the plastic spaghetti, they had difficulty later seeing the yarn as spaghetti. If they did not use the plastic spaghetti first, they were much more able to view the yarn as representing spaghetti. To Uttal, this aligned with his viewpoint that the type of manipulative, specifically how realistic matters. The teacher’s observations suggested that representing objects with manipulatives that are too realistic may prevent children from seeing that those objects can be represented in a different way.

Uttal believes that these types of conversations can guide research. So, he suggests, rather than designing a teaching method in a laboratory setting and then asking whether it would work in a classroom, researchers could first consider issues that arise in classrooms and then design corresponding experiments. To gain a deep understanding of mathematics learning in the classroom, Uttal believes that researchers need to engage in “a serious partnership with educators, teachers, principals, and parents.”

Conclusion

Uttal and the other researchers cited here do not dispute the fact that manipulatives serve a critical function in early mathematics education. Concrete objects can help children learn math concepts that might otherwise remain abstract and inaccessible. However, these researchers point out that children may not understand that a manipulative is intended to also represent something else—in other words, that it is a symbol. Thus, when using concrete objects in early mathematics education, they argue that teachers should consider whether children are able or likely to engage in dual representation—that is, understanding that manipulatives are both objects in and of themselves and symbols that represent mathematical concepts.
Selected References


