PCK for Preschool Mathematics: Teacher Knowledge and Math-Related Language Contribute to Children's Learning

Jennifer S. McCray

Erikson Institute

This manuscript is based on dissertation research completed under the direction of Dr. Jie-Qi Chen at Erikson Institute, and was funded by a Head Start Research Scholars Grant from the U.S. Department of Health and Human Services, Administration for Children and Families and a grant from the Chicago Public Schools.

Abstract

This study examines relationships between a new teacher interview designed to assess teachers' pedagogical content knowledge (PCK) for preschool mathematics, their math-related language, and school-year gains in mathematics ability scores by children in their classrooms. Twenty-six teachers and 141 children in Head Start programs in a large Midwestern city participated. Analysis using hierarchical linear modeling (HLM) finds significant relationships between scores on the new PCK Interview and gains in children's mathematics learning. In a new finding, frequency of teacher math-related language is only significantly related to child outcomes when delivered outside the large group setting; frequency of math-related language during "circle time" showed no association. A final HLM model combining PCK Interview scores and teacher math-related language outside the large group finds both contribute significantly and positively to child gains, suggesting that teacher PCK is not entirely mediated through math-related language as measured here.

PCK for Preschool Mathematics: Teacher Knowledge and Math-Related Language Contribute to Children's Learning

In 1988, Deborah Ball's dissertation demonstrated the importance of assessing teachers' pedagogical content knowledge (PCK) for teaching mathematics. Ball took Shulman's (1986) idea – that a particular kind of content knowledge is specially suited to teaching – and used it as the basis for a mathematics interview. In it, teachers were presented with elementary classroom scenarios in which a student misunderstood something, and teachers were asked how they would respond. Teachers with good PCK for teaching mathematics, Ball reasoned, would come up with representations of the mathematics that clarify meaning and guide future thinking. Shockingly few of the teachers Ball interviewed had even adequate responses to these scenarios; some teacher responses were simply incorrect. In a further application of Ball's interview, Ma (1999) interviewed Chinese teachers. Though these teachers were less educated than their U.S. counterparts, they offered rich, connected responses, demonstrating what Ma termed a "profound understanding of fundamental mathematics" (p. xxiv). Ball's interview accomplished two important things for mathematics education: first, it provided clear evidence that U.S. teachers' PCK for elementary mathematics teaching was lacking; and second, it suggested the types of content and pedagogy that might constitute good PCK, providing clear direction for elementary teacher education.

Unfortunately, Ball's interview and subsequent work related to PCK for elementary mathematics offer little to those of us wishing to better understand preschool mathematics and its effective teaching. There are profound differences in the knowledge bases of elementary and preschool mathematics; in particular, elementary-level mathematics uses written notation to support mathematical thinking, enabling the accomplishment of longer strings of procedures and acting as a sort of meta-cognitive crutch. Preschool children are active mathematics learners, but not ready to understand, let alone use, written arithmetic. Instead, three- and four-year-olds are primed to notice and explore the quantitative relationships in the world around them, and to begin using language and other forms of primary representation to crystallize information about pattern, shape, space, size, and number. Because of this, the preschool pedagogical task shifts from ensuring a connected, conceptual understanding of new mathematical procedures such as “moving the decimal over one place,” to helping young children recognize, name, and begin to experiment with the mathematics in their classroom environment and their own actions upon it. These particularly-preschool instructional tasks require a knowledge base distinct from the one Ball’s work has helped to identify.

Meanwhile, converging evidence from neuroscience, developmental psychology, mathematics and science educators, and early childhood professionals has helped to create an emerging sense that preschool mathematics education in the U.S. is in need of substantial improvement. It has become clear that early education is a more important teaching opportunity than was previously understood. Recent research clearly demonstrates the profound impact of early environmental influences on long-term brain development (Markezich, 1996), suggesting the potential of early education to create long-term benefits. Additionally, policymakers have become aware that more U.S. children are attending early care and education programs before formal schooling than ever before (U.S. Department of Education, 2000) and that good early education can have beneficial long-term learning outcomes and economic benefits (Barnett, 2008; Horton &

Bowman, 2002). Intervention that begins early has reported effects that extend into later years, apparently creating opportunity for greater educational and other gains in the long run (Bowman et al., 2001; Clements, Sarama, & DiBiase, 2004). Further, early intervention specifically focused on mathematics has been shown to have broad positive effects on student learning (Fuson, Smith, & Lo Cicero, 1997). These findings suggest that preschool education in the U.S. offers an important national opportunity, and that mathematics education merits special attention.

Other evidence suggests that the troubling tendency of American students to “lag behind” their counterparts in other industrialized nations on tests of mathematics (National Research Council, 1989, 1990; Schoenfeld, 1992) begins very early. Cross-national differences have been found in not only the early elementary school grades (e.g., Frase, 1997; Stevenson, 1987; Stevenson & Stigler, 1992) but also at the preschool level, where children from other developed and developing countries outperform their American counterparts on such beginning mathematics concepts as number words and early addition (e.g., Geary, Bow-Thomas, Fan, & Siegler, 1993; Ginsburg, Choi, Lopez, Netley, & Chi, 1997; Starkey, Klein, Chang, Dong, Pang, & Zhou, 1999). Growing evidence suggests that higher quality teaching of mathematics in the earliest years of a child’s education offers the best hope for improving mathematics achievement in the U.S. (Bowman et al., 2001; Clements, Sarama, & DiBiase, 2004).

While the education literature indicates that teacher quality is the most effective predictor of student achievement (Darling-Hammond, 2000; Just for the Kids and The Southeast Center for Teaching Quality, 2002; National Commission on Teaching & America’s Future, 1996; Rice, 2003), the quality of teaching at the preschool level is extremely variable (Copple, 2004), and mathematical teaching is no exception. A joint position statement on preschool math by the National Association for the Education of Young Children and the National Council of Teachers of Mathematics (NAEYC, 2005) is specific about the problem. The two organizations note the general lack of good teacher preparation in mathematics and point out that this under-preparation contributes to poor math-related attitudes among many early childhood teachers who lack confidence in mathematics anyway. While lack of confidence does not, in itself, prevent a teacher from teaching math, it can feed an unfortunate tendency to avoid math in the classroom. A recent contribution to preschool mathematics teacher training goes so far as to devote a section to this issue, noting "Math Anxiety – You Can Handle It" (Smith, 2001, p.2), and when surveyed (Carpenter, Fennema, Peterson, & Carey, 1988), both pre- and in-service teachers in early childhood classrooms expressed great reluctance to teach mathematics, making comments like "I don't do math." Describing early childhood educators, Copley notes "to them, mathematics is a difficult subject to teach and one area that they often ignore except for counting and simple arithmetic" (2004, p. 402). There is good reason to assume that the teaching of preschool mathematics could be improved.

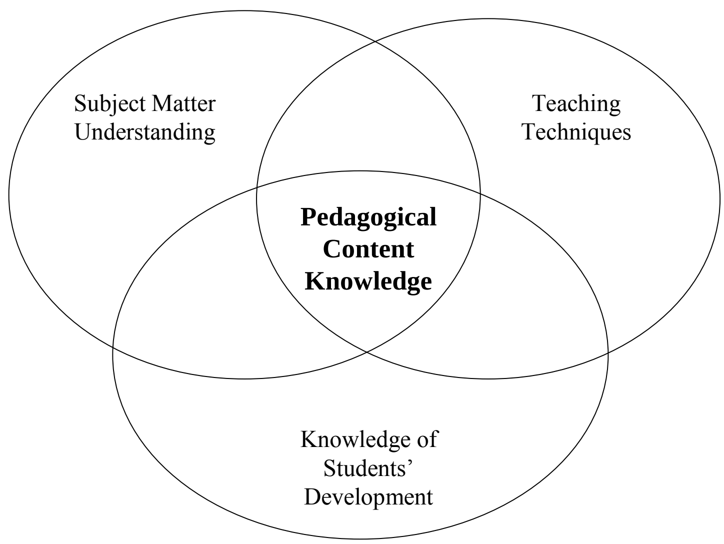
PCK for Preschool Mathematics – Construction of the Teacher Interview

In an attempt to better illuminate what preschool mathematics is and make clear the importance of teaching it knowledgeably and well, the author designed a new teacher interview to assess PCK for preschool mathematics. Literature on elementary mathematics education, preschool pedagogy, and mathematics content for very young children were consulted to create a tool that might allow teachers to demonstrate the knowledge that undergirds excellent preschool mathematics teaching. Where there was a lack of relevant preschool educational literature, ideas from cognitive developmental theory and recent research on quantitative development in early childhood were incorporated. The concept of pedagogical content knowledge (PCK), however, provided the guiding framework for the interview's content and construction.

According to Lee Shulman, who first identified pedagogical content knowledge (PCK) during his 1985 presidential address to the American Educational Research Association (Shulman, 1986), PCK is "a knowledge of subject matter for teaching which consists of an understanding of how to represent specific subject matter topics and issues appropriate to the diverse abilities and interest of learners" (Shulman & Grosman, 1988, p. 9). For example, PCK emphasizes knowledge of which content ideas are more central to the subject and how they connect to one another (subject matter understanding), appropriate examples for illustrating those concepts (teaching techniques), and awareness of how those concepts develop in the thinking of novices with differing levels of experience (knowledge of student development). In other words, knowledge of content, teaching practice, and student development are effectively combined where good PCK exists. See Figure 1 for a Venn diagram illustrating how these three bodies of knowledge overlap to construct PCK.

The new Preschool Mathematics PCK interview assessed here (see Appendix A) borrows Deborah Ball's 1988 innovation of situating interview questions within teaching scenarios and asking teachers to report what specific actions they would take in each situation (see also Study of Instructional Improvement, 2002). Using teaching scenarios rather than items solely about content knowledge contextualizes questions so they are more like the kinds of dilemmas teachers actually encounter in the classroom. This approach provides a mechanism for tapping content knowledge, pedagogy, and understanding of student learners simultaneously, acting as a better approximation of PCK than a purely content-related question could ever supply.

Figure 1. Pedagogical Content Knowledge (PCK)



Importantly, Ball's questions are also designed to assess teachers' ability to help students of elementary mathematics maintain and enhance connections between procedures and concepts in their mathematical thinking. The idea that there are two distinct types of mathematical knowledge – *procedural knowledge*, which encompasses the forms, rules, and processes that make it possible to accomplish mathematical tasks, and *conceptual knowledge*, which embodies richly connected ideas about the mathematical relationships between things and actions – is an old one. U.S. mathematics education has a long history of emphasizing first one of these types of knowledge, and then the other, with little effective result (see, e.g., McLellan & Dewey, 1895; Thorndike, 1922; Wheeler, 1939). It was not until the early 1980s that Resnick and Ford suggested that rather than trying to determine their relative importance to math education, the *relationships between* concepts and procedures ought to be emphasized (1981). In alignment with this thinking, Ball's interview pointedly asks elementary mathematics teachers to come up with real-world representations that can illustrate the usefulness and meaning of specific mathematics procedures.

For example, in one interview scenario, pre-service teachers were presented with the division statement 1 ¾ / ½ ("one and three quarters divided by one half"). Ball asked the pre-service teachers to "develop a representation – a story, a model, a picture, a real-world situation" of this division statement (Ball, 1988, p.16). An appropriate model of this statement would demonstrate that it will answer the question "how many halves are there in one and three quarters?" The effective teacher, Ball reasoned, would be able to describe a situation in which such a question is meaningful. As an example of a useful representation, Ball suggests "A recipe calls for ½ cup of butter. How many batches can one make if one has 1 ¾ cups butter? Answer: 3 ½ batches" (1988, p. 16). Interview questions such as this made clear which teachers had more of the kind of knowledge that would enable them to keep mathematical procedures meaningfully connected to the ideas they are meant to represent.

The concept-procedure literature in mathematics education is ambivalent about its own usefulness for preschool mathematics. Interestingly, it defines preschool "procedures" and elementary (and above) "procedures" extremely differently, though the ramifications of these differences are largely unexamined (see, e.g., Hiebert & Lefevre, 1986). In the elementary-based literature procedures are often algorithmic processes enacted upon symbols such as "borrowing from the 10s column," while in the preschool version of this analysis procedures are generally physical actions taken and enacted upon concrete objects. For example, in the concept-procedure literature, a toddler uses the "procedure" of placing a spoon in each teacup to construct a working concept of one-to-one correspondence (see Sinclair & Sinclair, 1986). From this perspective, very young children's mathematics concepts are completely dependent upon their procedures; mathematical knowledge is being created through action and has not yet been separated from the world of things. Sinclair and Sinclair comment "the young child cannot do without actual experience when logico-mathematical knowledge is in its beginnings" (1986, p. 63). As in elementary mathematics, connections between concepts and procedures in mathematical thinking are central; however, instead of being at risk of becoming separated from one another, perhaps they are at risk of not being firmly established in the first place.

The relatively sparse current literature on early mathematics education seems to support this perspective. Clements notes "although young children possess rich experiential knowledge, they do not have equal opportunities to bring this knowledge to an explicit level of awareness" and suggests the importance of differentiating between "the intuitive, implicit, conceptual foundation for later mathematics" and the subsequent elaboration that produces something more like conceptual math knowledge (2004, p. 11). Copley seems to agree, suggesting that conceptual learning requires something beyond actions taken upon concrete objects alone. She notes "Early childhood educators say that children learn by doing. The statement is true, but it represents only part of the picture. In reality a child learns by doing, talking, reflecting, discussing, observing, investigating, listening, and reasoning" (2000, p. 29).

If preschoolers can benefit from assistance in establishing and consolidating initial connections between their budding mathematical ideas and more generalized concepts, this provides an initial portrait of what good preschool mathematics teaching must be. Following the thinking of children as they interact with materials, recognizing the mathematical potential in their activities, and knowing how to comment on and extend their mathematics-related thinking must all be central. To provide opportunities for teachers to demonstrate these abilities, the new interview presents teachers with two classroom-based free play scenarios – one in the dramatic play area and one in the block corner. First, teachers read the scenarios to themselves, then the researcher reads them aloud, and finally, the researcher asks the teacher to do various things, including: identify specific math-related topics the children's play addresses; suggest a comment they might make to help the children think about/become more aware of the mathematics in their play; and propose a question they could ask that might encourage children to experiment with the mathematics in their play and extend their thinking. In this way, the interview is meant to elucidate preschool teachers' abilities to help young children construct and consolidate their initial mathematical concepts.

Each of the mathematics content strands proposed by the National Council of Teachers of Mathematics (NCTM) in its *Principles and Standards for School Mathematics* (2000) is represented in each scenario either through the use of specific materials, the comments children make during the scenario, or the problems children encounter and actions they take. For example, Scenario 1 presents teachers with the following text:

> Brittany and Jacob are playing in the dramatic play area and want to put their five babies to bed. There are no doll beds, so they make "cribs" out

> of three shoeboxes. Jacob says "but there aren't enough cribs." Brittany responds, "these babies are younger" picking out the three babies with no hair and setting them near the shoeboxes. She picks up the two babies with thick hair, says "these babies don't need to nap anymore," and sets them aside. Jacob says "OK, but this baby needs the most room" and puts the biggest bald baby in the biggest shoebox. Brittany watches him and then puts the medium-sized bald baby in the medium-sized shoebox and the smallest bald baby in the smallest shoebox. Jacob says "now go to sleep, babies." (Appendix A)

In this scenario, the small, medium, large relationships of the babies and shoeboxes represent a repeating pattern, while the idea of using a shoebox for a crib is an example of three dimensional geometric thinking. Recognizing that there "aren't enough cribs" requires one-to-one correspondence, which is foundational to number sense. When Brittany solves this dilemma by putting aside the two babies who have hair, she classifies and sorts a single set (babies) into two sets (bald babies and babies with hair). Measurement skills are activated when Jacob determines which baby needs the most room, and which shoebox offers it. In the interview, teachers who can see more mathematics in such play and generate effective comments that point it out and encourage its elaboration score more points.

A small group of experts in quantitative development and preschool pedagogy assessed the face validity of the interview, and contributed to a list of possible teacher responses. The list was used to construct response rubrics for each interview question which allow interviewers to check off key items as they are mentioned by teachers. Early piloting among three teachers indicated more and different responses than anticipated. Response rubrics were revised and resubmitted to the experts for comment; their comments were incorporated. Further piloting among six preschool teachers suggested the interview was sensitive enough to produce sufficient variability with a range of scores from 12 to 34 and a standard deviation of 7 (scores between 0 and 86 being possible).

Teacher Math-Related Language

The study also explores the relationship between this construction of preschool mathematics PCK and teachers' practices by examining teacher math-related language. Recent work (Ehrlich, 2007; Klibanoff, Levine, Huttenlocher, Vasilyeva, and Hedges, 2006) has found a positive association between frequency of preschool teachers' use of mathematics-related talk and gains in mathematics their students make during the school year. Assuming this finding would be replicated, coding systems from Klibanoff, et al., (2006) were applied to the classroom speech of the teachers interviewed so the predictive power of the interview could be compared to that of math-related language (see Appendix B).

The conceptualization of PCK above, however, suggested further differentiation of the relationship of math-related language to child outcomes might be possible. Teacher math-related language can appear at many times during a preschool day and in many different settings. It is often a part of the whole classroom meeting, or circle time, when preschoolers may count the number of children present or review the calendar and count off the date. It can also, however, be present when teachers work with a small group of children to complete a project, as in "each ladybug gets two eyes," or when teachers are supporting children's play, as in "four of those cups are little, and two are big." Since the representation of good mathematics pedagogy developed above suggested children develop mathematical concepts while operating directly on the world around them, it was theorized that circle time might not be the best opportunity for mathematical language to aid in this endeavor. Teacher math-related language offered while children are actively engaged in their own pursuits, however, could be differently received and processed because aligned with children's actions or mathematical "procedures." To assess whether pedagogical setting had any bearing on the relationship between math-related language and child learning, teacher speech data was gathered in both large group settings (circle time) and in free play and small group settings (non-circle time).

Research questions were:

1) Is there a significant association between teachers' PCK interview scores and gains children in their classrooms make?

2) Is there a significant association between teachers' math-related speech and gains children in their classrooms make (replicating the findings of others)? Is the strength of this association essentially the same for math-related speech during circle time and outside the circle time setting?

3) Is the relationship between PCK and child gains mediated by teacher math-related language, and if so, to what extent?

Methodology

<u>Participants</u>

Twenty-six Head Start teachers from a large urban area and 113 children from their classrooms with assessment scores at two time points participated. To be considered for the study, teachers had to work in classrooms in which English was not a second language for a majority of students and be "Head" teachers in their classrooms. Teachers were selected at random from among this group and recruited for participation;

14 CPS teachers agreed to participate. The author randomly selected non-CPS Head Start agencies to contact from a list provided by City of Chicago staff. Twelve non-CPS Head Start teachers agreed to participate. Participating teachers were given a $100 stipend and their classrooms received a set of picture books after teacher interviews were complete.

Classroom size ranged from 13 to 21 children, with a mean of 18 and a standard deviation of 2.5. While Head Start admits children between the ages of 3-0 and 4-11 at the start of the school year, on average, classrooms in the sample had five three-year-olds in September of 2006. Overall, 70% of the students in participating classrooms were African-American, 23% were Latino, and the remaining students were of other ethnic origin, including Brazilian, Caucasian, Chinese, Czech, German, Hmong, Polish, and Vietnamese. Taking overall classroom composition into account, more than half the classrooms served an all African-American population, several served a mostly-Latino population, and the remainder served a very diverse population. On average, 23% of students in these classrooms spoke English as a second language. The 26 classrooms were clustered within 19 sites. Fourteen sites held only one participating classroom, four sites had two participating classrooms, and one site held four.

In order to participate, children had to be between 3-4 and 5-0 years old, and be fluent in English, according to the teacher. When possible, six such children were randomly selected to participate from among those whose parents had consented in each classroom. One hundred forty-one children were assessed at T1 in the 26 classrooms that participated for the length of the study, or an average of 5.4 children per classroom. These children averaged 4 years and 4 months of age at first testing, and 76 of them, or 53.9%, were female. One hundred ten children, or 78.1%, were African-American, 26 were of Latino origin, and 5 were identified as Asian/Pacific Islander. Of these 141 children, only 113 had mathematics assessment scores at two time points (others having moved away or dropped out of their class). Only these children are included in the analysis, resulting in an average of 4.3 children per classroom.

The missing subjects were generally demographically similar to the entire sample. They averaged 4 years and 2 months of age, 90% came from English-only homes, and their ethnic make-up was strikingly consistent with the sample as a whole, with 80% of the group African-American, 16% Latino, and 4% Asian. They were, however, more likely to be female: 73.9% of these T1-only subjects were female, compared with 53.9% of the sample as a whole. Because of this, the remaining sample includes a higher percentage of male children (53%) than the original sample (46.1%).

Teacher Language Samples

Participating teachers' speech was tape-recorded and coded on one randomly selected day during the period from the beginning of January 2007 until end of April 2007. One hour of teacher speech was recorded and coded, always before noon, and included both "circle-time" (teacher-led large group) and the period immediately following; recording/coding continued until 60 minutes were captured. Activity after circle time consisted of combinations of free play, in which teachers supported children's activities in areas of their own choosing, and small-group activities, in which teachers met with a small group of children to guide them through a teacher-led set of activities. To facilitate recording, each teacher wore a wireless microphone, which broadcast a signal to be coded and recorded. Two researchers were present during recording to code teacher math-related language responses; a timer was utilized to prompt researchers to take turns coding in 10 minute intervals. Teachers were unaware of the mathematics-focus of the study when the language sample was recorded and coded.

The 60 minutes of coded teacher language began with the teacher's first address to the whole group of gathered children at the beginning of circle time. Coding categories from Klibanoff, et al. (2006) were used to code teacher math-related language (see Appendix B for category definitions). This coding system heavily emphasizes references to number and quantity – geometric and spatial math-related language is not explicitly represented. Because live coding of math-related language using this coding system had not been previously attempted, all language samples were recoded by the graduate student researcher using audiotapes, yielding an inter-rater reliability score of 96.4%; disagreements were settled by discussion.

Since language data was collected on only one day for each classroom, an examination of total math-related language instances was conducted to screen for outliers. That is, teachers who chose to emphasize mathematics on the one day of language recording far beyond what they would generally do needed to be accounted for, since the purpose of the language data was to find a representative sample of math-related language for each classroom. Using the commonly accepted formula of

$$< Q1 - (1.5 \times IQR) \quad \text{or} \quad > Q3 + (1.5 \times IQR)$$

in which Q1 = 1st quartile score, Q3 = 3rd quartile score, and IQR = Inter-quartile range (3rd quartile score – 1st quartile score), four teachers' frequency of math-related language scores were determined to be outliers; these four teachers and the data retrieved from within their classrooms (e.g., child mathematics gains) were removed from the data set for all further analysis.

Upon review of the audiotapes of these outlier teachers, it was discovered that one teacher spent the entire free play time going over number flash cards with children, another played a 25-minute game of number bingo, a third used almost 80% of the 60 minutes to play "find the number" with a small group of children (a game in which she said, for example, "where is the two?" and children picked out the correct numeral from a display), while the fourth spent her entire non-circle time working on ordinal relationships with a small group of children (saying, for example, "three, four, five...what comes next?"). While these activities may not represent unexpectedly advanced mathematics teaching, the dramatic impact they have on these teachers' language instances on the single day of recording and coding make the scores unlikely to be representative of their usual practice. Further, since these outlier classrooms represent the most math-related language delivered outside the circle time setting, their removal makes the assessment of non-circle time language as a predictor of child outcomes a more stringent one.

Test of Early Mathematics Ability

To assess gains Head Start children made in mathematics during the study period, the Test of Early Mathematics Ability, 3rd edition, (TEMA-3) (Ginsburg & Baroody, 2003) was administered twice – once in the fall and once in the spring. The TEMA-3 is a standardized, norm-referenced instrument designed to assess both informal (non-school taught) and formal mathematics achievements among 3 to 9 year old children. Basals and ceilings are utilized in the TEMA-3, and testing begins with an item corresponding to the child's age and ends when the child makes five consecutive errors. Children are asked to count the objects on a card, select the card with more objects and point to it, show a specific number of fingers, perform basic level mental addition and subtraction, and write numbers, among other things. TEMA-3 has two parallel forms with established test-retest reliability; children were randomly assigned Form A or Form B at T1 and received the alternate form at T2.

In 2002, the TEMA-2 (Ginsburg & Baroody, 1990) was reviewed as part of a workshop on "school readiness" sponsored by the U.S. Department of Health and Human Services, and was found to be "based on current theory and research…grounded in normative data…(and) a good predictor of mathematics achievement" (U.S. Department of Health and Human Services, 2002, p. 23). The normative sample for this assessment was representative of the national population in terms of sex, race (black, white, and other), region of the country, residence in an urban or rural community, and parent occupation (white-collar, blue-collar, service, farm, or other). Cronbach's Alpha for TEMA-3 math ability scores at age four is reported to be .93 for Form A and .95 for Form B, well within the most desirable range for reliability. Test-retest reliability is similarly high, with a delayed alternate-form coefficient of .93 (Ginsburg & Baroody, 2003).

Testing took place in a quiet setting near the child's classroom, often at a small table in the hallway just outside the room. Sometimes a separate room, down the hall from the classroom, was the option provided by the school. Assessments were conducted by graduate students in child development. Most TEMAs took approximately 15 minutes to administer, though assessment times ranged from about 2 minutes to almost 47 minutes, depending upon the child's ability and willingness to participate.

Results

Is There a Significant Association between Teachers' PCK Interview Scores and Gains Children in Their Classrooms Make?

Interview reliability. Because the Preschool Mathematics PCK Interview is a new measure, analyses were conducted to assess its internal reliability before examining its relationship to teacher practices. As a first measure of reliability, correlations were run between total Scenario scores. Scores for Scenarios 1 and 2, each meant to reflect PCK across mathematics content areas, correlated strongly ($r = .616$, $p < .001$). Utilizing sub-scenario scores, Cronbach's alpha for the items comprising these two Scenarios was a strong .76, representing an acceptable level of inter-item reliability (according to Nunnaly, 1978, the standard is .7.) Interviews were coded live, and fifteen (50 %) were coded simultaneously by two researchers. Inter-rater reliability was 92.8%; disagreements were settled by discussion and used an interview audiotape. Teachers scores ranged from 6 to 32 points, with an average score of 21.6 and a standard deviation of 5.8.

Children's gains in mathematics. At Time 1, the 91 children in non-outlier classrooms scored between 65 and 132 on the TEMA-3, with a mean score of 85.11 and a standard deviation of 12.43. This mean score is nearly 1 standard deviation below the population average of 100, indicating the sample children were doing noticeably worse on mathematics than their same-age peers nationally. At Time 2, scores had not changed that much overall. The children scored between 62 and 124, with a mean of 86.11 and a standard deviation of 14.74. In fact, six children showed no change in TEMA-3 score between time points, and 71.4% had scores that changed less than 10 standard points in either direction. Further, 53.8% of the sample showed either no change in score, or a negative change in score. Recall that the TEMA-3 is standardized for age so that simply getting the same test items correct from one session to the next is not enough to maintain the same standardized score. The lack of change reflected in some of these scores may represent an actual increase in knowledge and/or more items correct on the assessment while also indicating no movement relative to national averages for same-age peers. Regardless, this sample of Head Start children is not keeping up nationally, and by this measure, does not show an overall trend to improve its relative standing.

Relationship between interview scores and child gains. A 3-level Hierarchical Linear Model (HLM) was used to examine the relationship between PCK Interview scores and gains in child scores from T1 to T2 (see Table 1). HLM is generally considered appropriate whenever data has a nested structure, as it does in this study, since child outcomes are clustered by teacher and teachers are clustered by program sites. HLM offers several advantages over simple regression analysis in this case. First, while regression techniques allow examination of relationships between teacher PCK and child outcomes within classrooms, they neglect to account for variance related to program site. That is, because we know at which site each teacher is located, if we do not conduct analyses that account for site-level variance, we are not using all the available information. At least some of the variance in child outcomes is likely to be due to program site, rather than teacher; HLM allows us to attribute that variance accurately while assessing relationships between teacher PCK and child outcomes, yielding greater precision. Moreover, HLM allows the study to specifically examine the effects of program site and how those effects interact with teacher effects. That is, we can determine whether and how site characteristics might affect relationships between teacher language and child outcomes.

In effect, HLM creates one linear regression model at each level of analysis and then combines these equations into a single model. In this study, child variables (gains in TEMA-3 scores) are at Level 1, teacher variables (math-related language frequencies and interview scores) are at Level 2, and while no site variables are used as predictors, the use of a third level allows an examination of the effects of program sites. In this way, the modeled "levels" mimic the hierarchical clustering of the actual children, classrooms, and sites. In Level-1 of the example below, the gains of any particular child (TEMDIF) are modeled as the mean gains of all children in that classroom (π_0) plus some unique effect associated with the particular child whose gains are being modeled (e). The Level-2 model is meant to predict the mean gains of all the children in a given classroom (π_0) as the sum of the mean gains of all children at that particular program site (β_{00}) plus a unique effect associated with that particular classroom (r_0). Level-3 models the mean gains of all children at a given program site (β_{00}) as the grand mean of all children's gains (γ_{000}) plus a unique effect due to program site (u_{00}).

Level-1 Model

$$\text{TEMDIF} = \pi_0 + e$$

Level-2 Model

$$\pi_0 = \beta_{00} + r_0$$

Level-3 Model

$$\beta_{00} = \gamma_{000} + u_{00}$$

These 3 levels yield a combined model of:

$$TEMDIF = \gamma_{000} + u_{00} + r_0 + e$$

This explanatory model of gains on the TEMA-3 is sometimes called a fully unconditional model (see Raudenbush & Bryk, 2002, p. 24) because no predictors (or conditions) are specified. It is generally helpful to run such a model as a preliminary step because it provides baseline information about variability in the outcome variable (in this case, TEMDIF, or children's TEMA-3 gains) at each level of analysis. Results of the fully unconditional model for TEMDIF indicate that 19.6% of the variance between child gains can be attributed to differences at the classroom level. Since this study is focused on the impact of teachers' knowledge and practice upon child outcomes, this is the portion of the variance in child gains that is of interest. The fully unconditional model also indicates that this variance is significant, $X^2 = 25.86$ with 6 df ($p < .000$), but variance between sites is not, $X^2 = 24.31$ with 15 df ($p = 0.06$, *ns*). In other words, while classroom makes a significant contribution to variation among children's TEMA-3 gains, program site does not. Teachers (or something at the classroom level) appear to be having more of an impact upon children's mathematics learning than program sites.

To evaluate whether teacher PCK for preschool mathematics as measured by the new teacher interview can explain some of this classroom-level variance, interview scores were entered in the model as predictors at Level-2 (ITONETWO), resulting in the following model construction:

Level-1 Model

$$TEMDIF = \pi_0 + e$$

Level-2 Model

$$\pi_0 = \beta_{00} + \beta_{01}(ITONETWO) + r_0$$

Level-3 Model

$\beta_{00} = \gamma_{000} + u_{00}$

$\beta_{01} = \gamma_{010}$

When interview scores are entered into the model as a predictor, they significantly and positively predict gains in child outcomes; that is, the higher the teacher's PCK score, the greater the gains children in her classroom are likely to have made from T1 to T2 (see Table 1).

Table 1

Results of 3-Level HLM Models to Predict Child Gains on TEMA-3 (TEMDIF) Preschool Mathematics PCK Interview as a Predictor

Fixed Effect	Coefficient	Standard Error	T-ratio	*df*	*p*-value
For Intercept 1, π_0					
Intercept 2, β_{00}	-9.521734	4.032275	-2.361	15	0.032
ITONETWO, β_{01}	0.496883	0.180434	2.754	20	0.013

Is There a Significant Association between Teachers' Math-related Speech and Gains Children in Their Classrooms Make? Is the Strength of this Association Essentially the Same for Math-related Speech Both During Circle Time and Outside the Circle Time Setting?

Math-related language. Total frequency of math-related language for the 22 teachers ranged from 4 to 74 instances (*M* = 30.86, *S*D = 20.65). Circle time math-related language ranged from 0 to 50 instances (*M* = 18.95, *SD* = 16.01), and non-circle time math-related language ranged from 0 to 34 instances (*M* = 11.91, *SD* = 9.65). Circle and non-circle time frequencies did not correlate with one another ($r = .248$, $p = .133$, *ns*); that is, frequency of math-related language used during circle time did not significantly predict non-circle time frequency, and vice versa. Circle time ranged in length from almost 6 minutes to nearly 42 minutes ($M = 20.95$, $SD = 11.87$). As might be expected, teachers with longer circle-times used more instances of math-related language during that time ($r = .673$, $p < .000$). This relationship did not hold true for non-circle time, however ($r = -.144$, p = .261, *ns*); that is, teachers who spent more of the 60 minutes outside of circle time did not also tend to have more non-circle time math-related language instances. Frequency of math-related language during a particular pedagogical setting, then, cannot be interpreted as a measure of time on task. Additionally, the lack of correlation between amount of time spent and frequency of math-related language outside of circle time suggests this measure is more sensitive than the circle time measure to individual teacher variation.

<u>Relationship between teacher language and child outcomes</u>. Two 3-level hierarchical linear models (HLM) were conducted to examine the predictive power of teacher math-related language (see Table 2). In the first, to examine whether total frequency of math-related language during the 60 minute period accounts for variance at the classroom level, total math-related language (TOT) is entered into the otherwise fully unconditional model as a predictor. Surprisingly, total frequency of math-related language shows no significant relationship to child gains. As an alternative, the second model enters both circle time language (CIRCTOT) and non-circle time language (NONCIRC) simultaneously as Level-2 predictors. Accordingly, the model's results report the amount of variance explained by each of these variables after the other is

accounted for. This construction of the model is guided by the finding that circle time language and non-circle time language do not correlate with one another, and are therefore more likely to represent distinct contributions to variance among child outcomes.

Level-1 Model

$$\text{TEMDIF} = \pi_0 + e$$

Level-2 Model

$$\pi_0 = \beta_{00} + \beta_{01}(\text{CIRCTOT}) + \beta_{02}(\text{NONCIRC}) + r_0$$

Level-3 Model

$$\beta_{00} = \gamma_{000} + u_{00}$$

$$\beta_{01} = \gamma_{010}$$

$$\beta_{02} = \gamma_{020}$$

The results indicate that non-circle time math-related language has a significant positive relationship to gains in child outcomes, but circle-time language does not contribute significantly (see Table 2). To give a rough sense of the size of this effect, an increase of one math-related language instance during the 60 minutes coded was associated with a 0.26 increase in growth in TEMA-3 standard score ($p < .003$). Even after the variance attributed to this model is accounted for, child gains continue to differ significantly by teacher ($X^2 = 22.01$, $p < .000$), indicating there is still unexplained classroom-level variance.

Table 2

Results of 3-Level HLM Models to Predict Child Gains on TEMA-3 (TEMDIF)

Teacher Math-Related Language as a Predictor

Fixed Effect	Coefficient	Standard Error	T-ratio	*df*	*p*-value
Total Math-Related Language Instances (TOT) in 60 Minutes as a Predictor					
For Intercept 1, π_0					
Intercept 2, β_{00}	-1.460862	2.387392	-0.612	15	0.549
TOT, $\beta_{01,}$	0.083139	0.062128	1.338	20	0.196
Circle Time (CIRCTOT) and Non-Circle Time (NONCIRC) Math-Related Language in 60 Minutes as Predictors					
For Intercept 1, π_0					
Intercept 2, β_{00}	-1.939136	2.130691	-0.910	15	0.377
CIRCTOT, $\beta_{01,}$	-0.024915	0.070202	-0.355	19	0.726
NONCIRC, β_{02}	0.309822	0.109686	2.825	19	0.011

Is the Relationship between PCK and Child Outcomes Mediated by Teacher Math-related Language, and if so, to What Extent?

One final model was run to determine whether PCK and teacher math-related language have any distinct predictive power relative to child gains, or if instead, one of these contributors loses significance when variance due to the other is accounted for. In this model, PCK interview scores (ITONETWO), circle time language (CIRCTOT), and non-circle time language (NONCIRC) are all entered simultaneously as Level-2, or classroom-level, predictors. The resulting Combined PCK and Language Model (see

Table 3) indicates that both PCK and non-circle time language make independent contributions to the prediction of child outcomes, while circle time language remains non-predictive.

Table 3

Results of 3-Level HLM Model to Predict Child Gains on TEMA-3 (TEMDIF) Combined PCK and Language Model

Fixed Effect	Coefficient	Standard Error	T-ratio	*df*	*p*-value
For Intercept 1, π_0					
Intercept 2, β_{00}	-10.729137	3.382006	-3.172	15	0.007
CIRCTOT, $\beta_{01,}$	-0.102650	0.065273	-1.573	18	0.133
NONCIRC, β_{02}	0.261790	0.097992	2.672	18	0.016
ITONETWO, β_{03}	0.502984	0.171150	2.939	18	0.009

Discussion

The results indicate that higher teacher scores on the PCK for Preschool Mathematics Interview are significantly and positively associated with greater child gains on the TEMA-3 between T1 and T2. While the total frequency of math-related language during a 60 minute period showed no significant relationship to child gains, a subset of this variable, math-related language delivered outside the circle time setting, proved positively related. Math-related language delivered during circle time showed no significant association with child gains. Finally, the PCK interview score and teacher math-related language outside the circle time setting appear to act at least semi-independently as predictors of children's gains, since each remained significantly and positively associated with greater increases on the TEMA-3 even when variance attributed to the other was simultaneously accounted for.

Math-Related Language

While findings reported here generally support the idea that more teacher math-related language can positively influence children's mathematical learning – a conclusion that echoes those of Klibanoff et al. (2006) and Ehrlich (2007) – they also offer an additional insight. Specifically, it appears that pedagogical setting can interact with math-related language, impacting its effectiveness. In this study, only math-related language delivered outside of the large group, circle time setting shows a significant, positive relationship to child learning. More math-related language during circle time shows no association with children's improvement. Further, combining the two language measures dilutes the effect: Total frequency of math-related teacher language is not significantly related to child outcomes.

This effect of pedagogical setting is a new finding, not reported in previous literature on teacher math-related language. Klibanoff et al. (2006) collected language samples identical in form to those of this study – 60 minutes beginning with circle time – but made no distinction between pedagogical settings in their analysis. In their work, unlike this, teachers' total frequency of math-related language was a significant predictor of child learning. Ehrlich's (2007) study provides an even stronger contrast, since her language samples consisted exclusively of circle time, and again showed a significant and positive relationship. In Klibanoff et al., however, the effect size found is very small, since an increase of 25 instances of teacher math-related language during the 60 minute observation period would result in only .21 standard deviations of achievement gain

(2006). While Ehrlich finds a more sizeable effect, in which an increase of one language instance per minute results in 2.29 standardized points of gain in growth of the TEMA-3 score, this finding is only barely statistically significant ($p < .056$). Findings here for non-circle time language, in contrast, are of both reasonable size and high significance since an increase of only one math-related language instance during this period was associated with a 0.26 increase in growth in TEMA-3 standard score ($p < .002$). To provide a better comparison with Klibanoff, et al., an increase of 25 instances of teacher math-related language outside the circle time setting would result in .70 standard deviations of achievement gain, or more than three times as much. This comparison of effect sizes and significance levels suggests it is at least possible that the inclusion of circle time in language measures used by Klibanoff et al., (2006) and Ehrlich (2007) diluted the observable effects of language upon child outcomes.

It can also be argued that Klibanoff et al.'s (2006) and Ehrlich's (2007) classroom populations may not be a good match to those of this study in terms of SES and program quality. While Klibanoff et al. specifically examined SES and found no relationship to either teacher practices or child outcomes, they note their sample was weighted toward a high-SES population (2006). Of their 26 classrooms, only four were low SES, eight were middle-income, and 14 were high-SES. Hence, it is possible their sample was not representative enough to illuminate such differences, if they existed. Ehrlich's work directly assesses the effects of SES on the teacher language-child outcomes relationship by comparing results for Head Start and tuition-based preschool classrooms; she also finds no significant relationship between SES and the practice-outcomes relationship (2007). However, the Head Start classrooms that comprise her lower income group are located in Chicago Public Schools. This means they necessarily employ bachelor's level staff and offer a fairly rich and well-stocked classroom environment because of school funding and other institutional supports.

The classroom sample in this study includes not only Chicago Public School Head Start classrooms but also community-based Head Start classrooms, which are extremely variable in terms of teacher education and resources. At many of the community-based sites in this work, manipulatives were missing or broken, posters and visual displays were of poor quality, children had no or limited access to outdoor play spaces, and classroom space was limited. Often, it appeared that children spent much of their day supervised by assistant teachers who rarely held even associate's degrees; attention from the qualified teacher (whose language sample was recorded) was more limited. Community-based sites also frequently housed more than one Head Start classroom within a single large room, so the general noise and activity level was much higher than in most Chicago Public School classrooms. It can be legitimately argued that previous work on math-related teacher language did not look at this specific population of preschool classrooms. Less resource-rich classrooms, such as this study's overall sample includes, may make the relationship between math-related language and mathematics learning more vulnerable to interactions with pedagogical setting. Further work is required to evaluate this theory.

Post-hoc review of audiotapes indicates almost all the non-circle time language in this sample is delivered as an accompaniment to children's free play activity. Teachers comment on what children say, as in "Yes, you're right… you have *three* babies," involve themselves directly in children's play, as in "is there *one* more plate for me?" and extend children's thinking, as in "what would happen if we used *two* blocks instead of *three*?" In these types of situations, children are in the midst of play – manipulating materials, imagining scenarios, and constructing representations of their own. Teacher language serves to reflect, introduce, and extend the mathematical thinking embedded in these activities. Regardless of whether it is better overall pedagogy, children's self-directed engagement, teachers' individualized attention, or more likely, all these elements in concert that is at work in these non-circle time interactions, findings here provide persuasive evidence that teachers' math-related interactions with children during their free play have a powerful impact on their mathematical learning.

PCK Interview

Findings related to the interview suggest it functioned successfully as a measure of knowledge for teaching preschool mathematics. As expected, teachers with higher PCK Interview scores were more likely to teach children who made greater gains on the TEMA-3. While the interview may not be the only or best way to assess PCK for Preschool Mathematics, it appears to capture knowledge for teaching with a real relationship to learning among children. The ability of preschool teachers to see, comment on, and extend the mathematics they see in children's play is clearly associated with gains in their students' mathematical understanding. At least to some extent, the interview has successfully transferred the important work of elementary mathematics education researchers such as Hill, Rowan, and Ball (2005) to a preschool setting by providing a measure of mathematical content knowledge that is strongly associated with children's learning.

The final HLM model demonstrates that simultaneously accounting for variance in child gains associated with teacher math-related language (while it marginally reduces the size of the effect), actually increases the significance level of the relationship between PCK Interview Scores and child outcomes. This is especially surprising since the interview scenarios depict the very types of pedagogical settings in which non-circle time math-related language was collected. One might expect that a different measure of mathematics PCK that focused on, for instance, teachers' knowledge for planning large group activities, would be more likely to show a relationship to child outcomes that is not mediated by non-circle time math-related language. Because the interview is entirely focused on teacher-child interactions that occur outside circle time, it seems probable that the addition of non-circle time math-related language to the HLM model would subsume the contribution of PCK. Instead, we find each of these measures – one of teacher practice, and one of teacher knowledge – are significant predictors in their own right. In fact, one point on the interview contributes almost twice as much to variance in child gains as one instance of teacher math-related language (see Table 3). Frequency of math-related language cannot substitute for teacher knowledge, nor is it the case that frequency of math-related language is the only, or even major, way that PCK for preschool mathematics is expressed. In other words, while increasing the frequency of teacher talk about mathematics may be one way to augment children's mathematical understanding, knowing more about the specific mathematics they are discovering and how to help them see it appears equally and independently important.

While greater PCK predicted greater TEMA-3 gains, this does not speak to the overall quality of teachers' responses. When presented with scenario 1, in which children place babies in shoebox “cribs” of different sizes, teachers were quick to see the mathematics of measurement, but rarely saw that the cribs could elicit thinking about shape and spatial relationships. When asked to comment on the block play depicted in scenario 2, teachers tended to note the relevance of shape, but very seldom mentioned the importance of classifying types of blocks or using number to count blocks in an effort to build walls of the same length. The PCK of the study’s teachers appears to be supporting the mathematical learning that does occur in these classrooms; that does not mean, however, that it could not be radically improved. Particularly when we remember how far behind their same-age peers nationally children in these classrooms are, these teacher responses to the interview might be viewed as indications of how preschool educators could improve their mathematical PCK and thereby children’s mathematical learning.

Limitations and Future Research

This study has several important limitations. Because the findings reported here are associational, not causal, it is possible that some unmeasured factor, such as teacher attitude toward mathematics, is the real engine behind children’s learning. Controlled randomized design and replication of the results for both math-related language and the PCK Interview are needed to confirm causal relationships. Further, the PCK Interview is limited in scope, since it only directly addresses teacher knowledge used in free play settings. While the teacher interview scores may in fact be good indications of teacher knowledge for other kinds of preschool mathematics teaching, such as during circle time, we have no way of knowing this. Without some effective measure of PCK for leading large group explorations of preschool mathematics, it is unclear what contribution such knowledge might make. Finally, the Klibanoff, et al. coding system (2006) is constrained to teacher language that relates to number: measurement, spatial relations, geometry, and pattern are generally ignored. While this coding system is impressive in its robust relationship to gains in children's learning, its limitations in scope may impede its ability to capture much of teacher language that is of consequence. It may be that a broader coding system could show a greater mediating relationship between teacher math-related language and PCK for preschool mathematics.

Replication of the finding that pedagogical setting impacts the relationship between teacher math-related language and child outcomes is needed. If, in fact, pedagogical setting grows in influence on math-related teaching as SES or classroom quality declines, this is an important finding with many practical implications for intervention. There is some fairly robust work suggesting that circle time is not the most effective mechanism for addressing content in preschool classrooms generally and that children tend to learn more when engaged in free play (Montie, Claxton, & Lockhart 2007). However, given the universality of circle time in U.S. preschool education, more in-depth examination of large group teaching techniques and their effects across socio-economic groups may be in order. Further, circle time may serve important social and behavior management functions that allow other learning to take place; assessing for these differential types of effects could provide very useful information for early childhood educators.

Conclusion

The concept-procedure analysis of mathematical knowledge helps illuminate the central aim of effective mathematics education: namely, imparting a system of mathematical thinking in which procedures and concepts are well-connected. This educational goal works as well for preschool children as for elementary school children. The difference is that mathematical procedures at these two levels of development take very different forms. While elementary mathematics teachers must strive to impart notation and algorithms (procedures) in a manner that emphasizes their meaningful relationship to mathematical ideas about the world (concepts), preschool teachers must work to provide language that captures key relational abstractions (concepts) just as children encounter them through their actions upon objects (procedures). For preschoolers, effective mathematics teaching must acknowledge the central importance of their initial construction of mathematical concepts from experiences in the world and so focus on naming and making that mathematics explicit.

Clements recommends mathematical teaching that "involves reinventing, redescribing, reorganizing, quantifying, structuring, abstracting, generalizing, and refining that which is first understood on an intuitive and informal level" (2004, p. 12). Like Rogoff's cognitive "apprenticeships" (1990), this type of teaching can be thought of as a socialization process, in which young children, led by expert adults, learn to see and think about the world in terms of quantitative relationships for the first time. In turn, the meaningfully explicit mathematics of the preschooler should serve as a solid foundation for the application of numeracy and notation that constitute elementary school arithmetic. The findings reported here support the view that making mathematical thinking more explicit and meaningful by connecting children's actions to mathematical language can have a lasting impact on their learning during a single school year, providing a better foundation for later mathematical learning.

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Appendix A

PCK for Preschool Mathematics Teacher Interview

Different Kinds of Math

- NUMBER SENSE
- PATTERNS
- OPERATIONS
- MEASUREMENT
- SHAPE
- SPATIAL RELATIONSHIPS
- CLASSIFICATION

Scored By ______________________ ID ______________ Date ______________ File Name ____________

This is an interview about how you think about preschool math. It has four classroom scenes. I'm going to ask you to read each scene to yourself, then I'll read it with you, and then I'll ask you some questions about it. OK?

In the first two scenes, children are doing free play. In these scenes I want you to read through them and see if you can see any math in their play. When I say math, I mean things like:

- Number sense (for example, counting, number use, 1-to-1 correspondence)
- Patterns
- Operations (like combining, taking away)
- Measurement (like comparing, estimating, using units)
- Shape
- Spatial relationships
- Classification (for example, sorting, matching)

Let's start with Scene One now. Please read it through to yourself looking for math, and then I will read it with you before I ask you any questions about it.

(TEACHER READS.)

Now let's read it together.

Scene One

Brittany and Jacob are playing in the dramatic play area and want to put their 5 babies to bed. There are no doll beds, so they make "cribs" out of three shoeboxes. Jacob says "but there aren't enough cribs." Brittany responds, "these babies are younger" picking out the three babies with no hair and setting them near the shoeboxes. She picks up the two babies with thick hair, says "these babies don't need to nap anymore," and sets them aside. Jacob says "OK, but this baby needs the most room" and puts the biggest bald baby in the biggest shoebox. Brittany watches him and then puts the medium-sized bald baby in the medium-sized shoebox and the smallest bald baby in the smallest shoebox. Jacob says "now go to sleep, babies."

<u>Scene One</u>: Brittany and Jacob Play with Babies

<u>VOLUNTEERED RESPONSES</u>

Where do you see any math in this play? [**Probe**: What part of the children's play has math in it?] Some people see only one example of math, while some people see more. Can you see any other math in this play? [**Probe**: What *other* math do you see in this play?]

PLAY EXAMPLE **[+1]** Where do you see any math in this play? [**Lead to Example**]: What is it the kids are doing that makes you think of ________ (content mentioned)?]	HOW IS IT MATH? **[+2 when tied to example]** [**From Example**]: How is that mathematical? [**From List**]: Why is it an example of _______ (content mentioned)?	LIST **[+2 when tied to ex]** If you had to describe this math using these terms (provide list), how would you describe it?	SCORES
□ Baby-to-shoebox by size order	□ rule that repeats □ order by size □ monotonic seriation	□ PATTERNS	[0-5]
□ Shoebox = crib	□ shape/space match	□ SHAPE	[0-5]
□ Babies INSIDE shoebox	□ "crib" encloses □ relationships of location/ btwn objects	□ SPATIAL RELATIONS	[0-5]
□ Babies/shoeboxes differ in size	□ comparison by size	□ MEASUREMENT	[0-5]
□ Sort by hair = age	□ logic / similarity □ grouping / which go together	□ CLASSIFICATION	[0-5]
□ 3 cribs hold 3 babies	□ counting, number use □ 1-to-1 correspondence	□ NUMBER SENSE	[0-5]
	□ compare/see amount/number	□ MEASUREMENT	[0-4]
□ Put two babies aside	□ take away right amount/number	□ OPERATIONS	[0-5]
□ Other [0 points possible]		**TOTAL VOLUNTEERED**	[0-39]

Scene One: Brittany and Jacob Play with Babies

PROMPTED RESPONSES

[For those content areas *not selected previously*]…
Do you see any use of _____________ (content area) in this play? Please answer either yes, no, or not sure.

[If "yes" to above]…
Where do you see _____________ (content area) in this play?

[If "not sure" to above]…
Where do you think you might see _______________ (content area) in this play?

Prompted?	Content Area	Response	Specific Play Example	Score
□	NUMBER SENSE	□ yes/not sure **[+2 with corresponding ex only]** □ no	□ 3 cribs hold 3 babies	[0-2]
□	PATTERNS	□ yes/not sure **[+2 with corresponding ex only]** □ no	□ Baby-to-shoebox by size order	[0-2]
□	OPERATIONS	□ yes/not sure **[+2 with corresponding ex only]** □ no	□ Put two babies aside	[0-2]
□	MEASUREMENT	□ yes/not sure **[+2 with corresponding ex only]** □ no	□ Babies/shoeboxes differ in size	[0-2]
		□ yes/not sure **[+2 with corresponding ex only]** □ no	□ 3 cribs hold 3 babies	[0-2]
□	SHAPE	□ yes/not sure **[+2 with corresponding ex only]** □ no	□ Shoebox = crib	[0-2]
□	SPATIAL RELATIONSHIPS	□ yes/not sure **[+2 with corresponding ex only]** □ no	□ Babies INSIDE shoebox	[0-2]
□	CLASSIFICATION	□ yes/not sure **[+2 with corresponding ex only]** □ no	□ Sort by hair = age	[0-2]
			TOTAL SCORE PROMPTED	[0-16]

Scene One: Brittany and Jacob Play with Babies

ELABORATIONS

Can you think of a *comment you could make* to help the children see some of the math in their play? [**Probe**: What could you say about what the kids have done to help them understand that there is math in their play?]

□ MATH-FOCUSED
□ PLAY-RELATED

[+2 if both checked above]: _______________

(Show list) Which *math topic* on this list do you believe this comment would help the children see or think about? [**Probe**: What specific math ideas would your comment help the children think about?]

[+2 if connects to comment above]: _______________

Can you think of a *question you could ask* to help the children find out more about the math in their play? [**Probe**: What could you ask the kids that would encourage them to think further about or experiment with the math in their play?]

□ MATH-FOCUSED
□ QUESTION

[+2 if both checked above]: _______________

(Show list) Which *math topic* on this list would this question help the children explore? [**Probe**: What math ideas would this question help the children work on?]

[+2 if connects to comment above]: _______________

TOTAL ELABORATIONS (0-8): _______________

Now let's do Scene Two. Please read it to yourself first.

(TEACHER READS.)

Now let's read it together.

Scene Two

Brandon and Tyra are playing with unit blocks and want to build a cage for a mama elephant and her two babies. Tyra builds the first two sides of the cage, set up at a right angle to each other, and using two unit blocks for each side. Brandon sets up the third cage side, but uses one unit block and a half unit block instead of two full units. When Brandon tries to finish the cage by building the 4th side, he sees that it doesn't hit the 1st side exactly at the corner. He says, "hey, it doesn't work…I'll fix it." He adds another half unit block to his 3rd side and he and Tyra finish the cage together. Tyra and Brandon place the three elephants inside.

<u>Scene Two</u>: Brandon and Tyra build a cage

VOLUNTEERED RESPONSES

Where do you see any math in this play? [**Probe**: What part of the children's play has math in it?.] Some people see only one example of math, while some people see more. Can you see any other math in this play? [**Probe**: What *other* math do you see in this play?]

PLAY EXAMPLE **[+1]** Where do you see any math in this play? [**Lead to Example**]: What is it the kids are doing that makes you think of _________ (content mentioned)?]	HOW IS IT MATH? **[+1 when tied to example]** **[From Example]**: How is that mathematical? **[From List]**: Why is it an example of _______ (content mentioned)?	LIST **[+1 when tied to ex]** If you had to describe this math using these terms (provide list), how would you describe it?	SCORES
□ two blocks each side	□ repetition of number	□ PATTERNS	(0-3)
	□ counting blocks used	□ NUMBER SENSE	(0-2)
□ Cage = rectangle	□ seeing/matching shapes	□ SHAPE	(0-3)
□ Can make rectangle	□ rt angles/parallel lines □ ends/corners meet	□ SHAPE	(0-3)
□ Select rectangular blocks	□ best for building wall □ match to wall shape	□ SHAPE	(0-3)
□ **Elephants INSIDE cage**	□ relations in space/btwn objects □ "cage" encloses	□ SPATIAL RELATIONS	(0-3)
□ 3^{RD} side is shorter	□ compare length	□ MEASUREMENT	(0-3)
□ Combine 2 sm = 1 lg	□ estimate difference/relation □ unit use	□ MEASUREMENT	(0-3)
	□ adding makes more	□ OPERATIONS	(0-2)
□ Size of cage fits elephants	□ size/space estimate	□ MEASUREMENT	(0-3)
□ types of blocks	□ seeing diff types of blocks □ matching block types	□ CLASSIFICATION	(0-3)
□ Other [0 points possible]		**TOTAL VOLUNTEERED**	(0-31)

Scene Two: Brandon and Tyra build a cage

PROMPTED RESPONSES
[For those content areas *not selected previously*]…
Do you see any use of _____________ (content area) in this play? Please answer either yes, no, or not sure.

[If "yes" to above]…
Where do you see _____________ (content area) in this play?

[If "not sure" to above]…
Where do you think you might see _______________ (content area) in this play?

Prompted?	Content Area	Response	Specific Play Example	Score
□	NUMBER SENSE	□ yes/not sure **[+1 with corresponding ex only]** □ no	□ two blocks each side	[0-1]
□	PATTERNS	□ yes/not sure **[+1 with corresponding ex only]** □ no	□ two blocks each side	[0-1]
□	OPERATIONS	□ yes/not sure **[+1 with corresponding ex only]** □ no	□ combine 2 sm = 1 lg	[0-1]
□	MEASUREMENT	□ yes/not sure **[+1 with corresponding ex only]** □ no	□ combine 2 sm = 1 lg	[0-1]
		□ yes/not sure **[+1 with corresponding ex only]** □ no	□ 3rd side shorter	[0-1]
		□ yes/not sure **[+1 with corresponding ex only]** □ no	□ cage size fits elephants	[0-1]
□	SHAPE	□ yes/not sure **[+1 with corresponding ex only]** □ no	□ cage = rectangle	[0-1]
		□ yes/not sure **[+1 with corresponding ex only]** □ no	□ can make rectangle	[0-1]
		□ yes/not sure **[+1 with corresponding ex only]** □ no	□ select rectangular blocks	[0-1]
□	SPATIAL RELATIONSHIPS	□ yes/not sure **[+1 with corresponding ex only]** □ no	□ Elephants INSIDE cage	[0-1]
□	CLASSIFICATION	□ yes/not sure **[+1 with corresponding ex only]** □ no	□ 2 types of blocks	[0-1]
			TOTAL SCORE PROMPTED	[0-11]

Scene Two: Brandon and Tyra build a cage

ELABORATIONS

Can you think of a *comment you could make* to help the children see some of the math in their play? [**Probe**: What could you say about what the kids have done to help them understand that there is math in their play?]

□ MATH-FOCUSED
□ PLAY-RELATED

[+2 if both checked above]: _______________

(Show list) Which *math topic* on this list do you believe this comment would help the children see or think about? [**Probe**: What specific math ideas would your comment help the children think about?]

[+2 if connects to comment above]: _______________

Can you think of a *question you could ask* to help the children find out more about the math in their play? [**Probe**: What could you ask the kids that would encourage them to think further about or experiment with the math in their play?]

□ MATH-FOCUSED
□ QUESTION

[+2 if both checked above]: _______________

(Show list) Which *math topic* on this list would this question help the children explore? [**Probe**: What math ideas would this question help the children work on?]

[+2 if connects to comment above]: _______________

TOTAL ELABORATIONS (0-8): _______________

Appendix B

Math-Related Language Coding System from Klibanoff et al., 2006

1) Counting – encompasses both reciting counting words and counting objects in sets.
2) Cardinality – involves stating (or asking for) the number of things in a set without counting them. If cardinality is used to reinforce counting, it is coded as a separate instance, e.g., "One, two, three. There are 3 books" would be coded as two instances, one of counting and one of cardinality.
3) Equivalence – encompasses statements describing a quantitative match, either of number or of amount, between two or more entities. These include: (a) one-to-one mapping, e.g., each child gets one cracker; (b) one-to-many mapping, e.g., each group has four children; (c) stating that two amounts or sets are the same.
4) Nonequivalence – encompasses statements of two or more entities being unequal, whether referring to (a) unspecified amounts, e.g., "Who has the most?" (b) one amount specified and the other(s) unspecified, e.g., "Oh no, you have more than twelve teeth" (c) all relevant amounts specified, e.g., "Seven people said yes, ten people said no. Did more people say yes or say no?"
5) Number symbols - coded if utterances include instances in which a teacher labels a written number symbol, or asks a child to identify, write, or find a number symbol, e.g., "3" in a stack of cards with printed numbers.
6) Conventional nominatives – numbers used as labels for things or dates.

7) Ordering – instances of referral to a sequence with explicit reference to more than one entity or set. Note that reciting a list of number words in order would not be coded as ordering but rather as counting.

8) Calculation – includes cases in which a teacher performs a calculation or asks a child to solve a calculation problem.

9) Place-holding – encompasses any input that refers to place value: ones, tens, hundreds, etc., including, but not limited to, the decomposition of (at least) two-digit numbers.